# TRACE Back from the Future: A Probabilistic Reasoning Approach to Controllable Language Generation

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## Abstract

As large language models (LMs) advance, there is an increasing need to control their outputs to align with human values (e.g., detoxification) or desired attributes (e.g., personalization, topic). However, autoregressive models focus on next-token predictions and struggle with global properties that require looking ahead. Existing solutions either post-train LMs for each new attribute—expensive and inflexible—or approximate the Expected Attribute Probability (EAP) of future sequences by sampling or training, which is slow and unreliable for rare attributes. We introduce **TRACE** (**T**ractable Probabilistic **R**easoning for **A**daptable **C**ontrollable g**E**neration), a novel framework that efficiently computes EAP and adapts to new attributes through tractable *probabilistic reasoning* and lightweight *control*. TRACE distills a Hidden Markov Model (HMM) from an LM and pairs it with a small classifier to estimate attribute probabilities, enabling exact EAP computation over the HMM's predicted futures. This EAP is then used to reweigh the LM's next-token probabilities for globally compliant continuations. Empirically, TRACE achieves state-of-the-art detoxification results with only 10% decoding overhead, yields 76 low-resource personalized LMs within seconds, and seamlessly extends to composite attributes.

## 1. Introduction

As large language models (LMs) become more ubiquitous in commercial products and daily life, there is a growing need to *control* their outputs. There is a large body of research in LM alignment with objectives such as detoxification (Gehman et al., 2020; Xu et al., 2021), where abundant data and benchmarks exist. Meanwhile, there is growing interest involving specialized or personal attributes under low-data conditions (Adiwardana et al., 2020; Xu et al., 2021), as well as compositional attributes for creating complex or novel outputs that rarely appear during standard training (Liu et al., 2023).

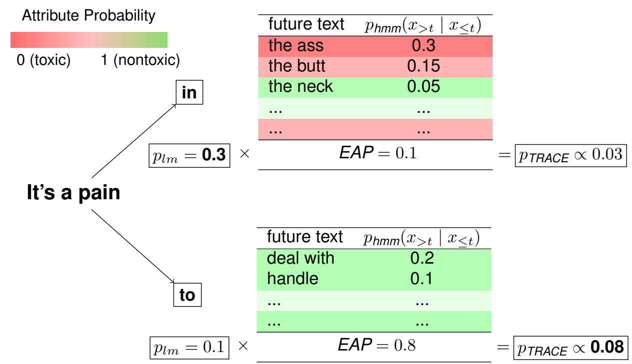


*Figure 1.* TRACE reweighs LM next token probabilities by "lookahead," computing the Expected Attribute Probability (EAP), $\sum_{x_{>t}} p(s \mid x_{\leq t}, x_{>t}) \cdot p_{hmm}(x_{>t} \mid x_{\leq t})$, using an HMM to tractably compute the expectation of a probabilistic classifier $s$.

Controllable text generation is challenging because most language models are autoregressive, generating each token solely from its predecessors without looking ahead—yet many attributes depend on the *entire* text. One line of solutions modifies the base model's distribution via fine-tuning or post-training (e.g., RL, RLHF) (Gururangan et al., 2020; Rafailov et al., 2024; Schulman et al., 2017), but these approaches can be highly expensive and risk degrading fluency or diversity (Kumar et al., 2022). Thus, there is a need for more lightweight solutions that can directly leverage pretrained LMs by changing the decoding strategy.

Fundamentally, we argue that decoding-based controllable generation is a *probabilistic reasoning* task: we wish to sample from the language model's distribution *conditional on some attribute*. For example, some approaches utilize sampling to find attribute-consistent generations (Yang & Klein, 2021; Tu et al., 2024; Chakraborty et al., 2024; Mudgal et al., 2024), but this can be computationally expensive. Other methods train expensive LM-based discriminators to predict satisfaction of an attribute given a partial text sequence (Krause et al., 2021; Meng et al., 2022; Liu et al.,

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2021), but this requires large amounts of data and training time. Both struggle with rare attributes due to limited data and high variance.

In this work, we propose a framework for controllable generation that utilizes *tractable models* to approximate the probability of satisfying an attribute given a partial sequence. This approach avoids costly sampling methods and eliminates the need for *attribute-specific fine-tuning or retraining* of the base language model (LM). First, we perform a one-time distillation of a tractable Hidden Markov Model (HMM) to approximate the base LM. Next, for each desired attribute, we train a log-linear classifier to estimate the probability of satisfying a target attribute given the entire text. During decoding, we use Bayesian conditioning to reweight next-token probabilities based on the likelihood that future sequences—predicted by the HMM—comply with these attributes, which we refer to as Expected Attribute Probability (EAP) (Figure 1). Crucially, the combination of the tractable HMM and log-linear classifier enables us to compute EAP efficiently and exactly over all of the exponentially many future sequences (over the HMM state space).

Our method, **T**ractable probabilistic **R**easoning for **A**daptable **C**onstrained g**E**neration (**TRACE**), constitutes a uniquely *lightweight* solution that enforces control with almost zero decoding-time overhead over the base LM. It further decouples generative model *training* from *control*, eliminating the need to retrain the LM or HMM for new objectives. Adapting to novel or rare attributes merely requires training a small classifier in seconds. Handling compositions of attributes is also straightforward by multiplying the EAP of each attribute during decoding.

We evaluate TRACE on three important tasks: *(1) Detoxification*: TRACE outperforms expensive RL, training- and sampling-based baselines on GPT2-large and Gemma 2B, taking only $\sim 1.1\times$ time per-token to standard decoding. *(2) Personalized LLMs*: TRACE adapts to 76 distinct characters in about three seconds each, outperforming prompting approaches with only a few hundred training samples. *(3) Compositional Attributes*: We show how to easily generate conditional on compositions of attributes, which may be anti-correlated such as texts that are both *political* and *non-toxic*. Overall, **TRACE** is a simple, lightweight controllable generation approach that achieves state-of-the-art detoxification performance, extends to low-resource and composite text generation, and scales well to modern LLMs.

## 2. Related Work

Controllable text generation methods can be broadly categorized based on whether they modify the underlying language model through training or alter the decoding process of a pre-trained model.

### 2.1. Training Methods

One line of approaches modifies the base language model's (LM) parameters, typically through **fine-tuning or reinforcement learning**, to instill desired attributes directly into the model's distribution. **DAPT** (Gururangan et al., 2020) fine-tunes the base LM on domain-specific data. **PPO** (Schulman et al., 2017) uses a reward model and policy gradients to fine-tune the LM towards desired behaviors like non-toxicity, while **DPO** (Lee et al., 2024) aligns the model using pairwise preferences without an explicit reward model. Similarly, **Quark** (Lu et al., 2022) uses an RL-like procedure conditioned on a learned reward token. A major drawback of these methods is the need for substantial data and costly retraining of the large LM for *each* new attribute or set of attributes. Furthermore, modifying the base LM risks degrading its general fluency and diversity (Kumar et al., 2022).

### 2.2. Decoding Methods

An alternative direction focuses on modifying the decoding process of a fixed, pre-trained LM to steer generation towards desired attributes, often by incorporating an estimate of the Expected Attribute Probability (EAP) of future text.

**Training Discriminators.** As directly computing the EAP requires summing over exponentially many future sequences, one approach trains auxiliary models (discriminators or guides) to predict the likelihood of satisfying an attribute based only on the *partial* sequence generated so far. **FUDGE** (Yang & Klein, 2021) trains a discriminator that predicts EAP to modify next-token probabilities. **DExperts** (Liu et al., 2021) combines pre-trained "expert" and "anti-expert" LMs, while **GeDi** (Krause et al., 2021) trains smaller attribute-conditional LMs (guides) to reweigh the base LM's distribution via Bayes' rule. **LiSeCo** (Cheng et al., 2024) uses a learned linear probe in latent space. A fundamental challenge here is that accurately predicting the final attribute satisfaction from a prefix requires the auxiliary model to implicitly reason about likely continuations and their properties, effectively needing sophisticated lookahead capabilities that are difficult to train reliably. Furthermore, these methods still typically require training a separate, often large, auxiliary model for each attribute.

**Sampling.** Other methods incorporate EAP by sampling future sequences to estimate expected outcomes. Some perform limited token-level lookahead (Mudgal et al., 2024; Chakraborty et al., 2024; Tu et al., 2024), managing complexity by restricting the search space. Others employ MCMC-style sampling on entire sequences using energy-based models (e.g., **MuCoLa** (Kumar et al., 2022), **Mix and Match** (Mireshghallah et al., 2022), **COLD** (Qin et al., 2022)). Both approaches incur significant computational overhead during decoding and the resulting EAP estimates can have high variance, especially for rare attributes.

While all methods attempting EAP-based control must approximate the true (intractable) EAP under the base LM, the *nature* of the approximation differs significantly. Discriminator/guide-based and sampling-based methods directly approximate the EAP value during decoding, often requiring complex per-attribute training or incurring high variance and computational cost at generation time. In contrast, TRACE shifts the primary approximation to the one-time HMM distillation step ($p_{\text{hmm}} \approx p_{\text{lm}}$). Conditioned on this distilled HMM, TRACE then performs an *exact and tractable* computation of the EAP ($p_{\text{hmm}}(s \mid x_{<t}, x_t)$) relative to the HMM distribution, as detailed in Section 4.2. This design choice—approximating the LM via a one-time HMM distillation—enables efficient, adaptable, and low-variance EAP calculation during decoding, avoiding per-attribute retraining or costly sampling.

### 2.3. Control via Tractable Models

Tractable probabilistic models (Choi et al., 2020) such as HMMs enable efficient computation of various quantities such as marginals and the probability of satisfying logical constraints, while such computations on autoregressive models are often provably hard even to approximate (Roth, 1996). They have been effectively used, for instance, to enforce adherence to *logical* or *lexical* requirements in generative modeling (Liu et al., 2024a;c). In the context of language modeling, Ahmed et al. (2023) proposed using a local tractable approximation for training autoregressive models to satisfy logical constraints. Meanwhile, Zhang et al. (2023; 2024) proposed utilizing HMM structures specifically to enforce logical constraints such as those given by deterministic finite automata (DFA).

While powerful for such formally specifiable constraints, this approach does not readily extend to controlling generation based on high-level *semantic* properties like style, safety, or persona, which lack simple logical definitions and depend on the overall meaning of the text. Our work, TRACE, bridges this gap by employing the HMM uniquely for *semantic* control, enabling efficient *probabilistic reasoning* about semantic attributes (via EAP computation) rather than enforcing logical rules.

## 3. Preliminaries

### 3.1. Controllable Generation

We consider generating a text (sequence of tokens) $x_{1:n}$ of length $n$ from a large language model (LM). In controllable generation, the goal is to generate text from the LM conditional on some attribute $s$, such as nontoxicity. We assume that the attribute can be measured by some probabilistic classifier $p(s|x_{1:n}) \in [0, 1]$ representing the probability (or degree) of satisfaction of the attribute given the full text.

Let us denote the base LM distribution by $p_{lm}(x_{1:n})$. Then the joint distribution over the text $x_{1:n}$ and attribute $s$ is

$$p_{lm}(x_{1:n}, s) = p_{lm}(x_{1:n})p(s|x_{1:n}). \tag{1}$$

Our goal is then to generate from the *conditional* distribution $p_{lm}(x_{1:n} \mid s)$. This conditional distribution can be decomposed autoregressively as:

$$p_{lm}(x_{1:n} \mid s) \;=\; \prod_{t=1}^{n} p_{lm}(x_t \mid x_{<t}, s)\,. \tag{2}$$

However, sampling from the conditional next-token distribution $p_{lm}(x_t \mid x_{<t}, s)$ is generally intractable. Using Bayes' rule, this is given by:

$$p_{lm}(x_t \mid x_{<t}, s) \propto p_{lm}(x_t \mid x_{<t}) \cdot p_{lm}(s \mid x_t, x_{<t}). \tag{3}$$

The first term is simply the LM next token distribution. The second term is the probability of satisfying the attribute $s$; this requires summing over all possible continuations $x_{>t}$, which is exponential in sequence length:

$$p_{lm}(s \mid x_t, x_{<t}) \;=\; \sum_{x_{>t}} p(s|x_{\leq t}, x_{>t})\; p_{lm}(x_{>t} \mid x_{\leq t}).$$

Since the classifier is probabilistic, we will also call this the *expected attribute probability* (EAP). The EAP is used to reweight the possible token generations $x_t$ according to how likely they are to result in a text that eventually satisfies the desired attribute. Several existing approaches effectively aim to approximate this *computationally hard* sum. For example, GeDi and DExperts train step-wise discriminators to guide the LM. Other approaches, such as Controlled Decoding and MuCoLa sample future sequences for lookahead. In this work, we aim for a tractable way to incorporate future-sequence information without expensive sampling or retraining, using Hidden Markov Models.

### 3.2. Hidden Markov Models

Hidden Markov models (HMM) specify a joint distribution over a set of latent variables $z_{1:n}$ and observed variables $x_{1:n}$, as

$$p(x_{1:n}, z_{1:n}) = p(z_1)p(x_1 \mid z_1) \prod_{t=2}^{n} p(z_t \mid z_{t-1})\, p(x_t \mid z_t). \tag{4}$$

For language modeling, each $z_t$ takes values in $\{0, \ldots, h-1\}$, where $h$ is the *hidden state size*, while the observed variables $x_t$ are tokens taking values in $\{0, \ldots, V-1\}$, where $V$ is the vocabulary size. The (homogeneous) HMM has $h^2$ parameters for the transition matrix $p(z_t|z_{t-1})$, $hV$

parameters for the emission matrix $p(x_t|z_t)$, and $h$ parameters for the initial hidden state distribution $p(z_1)$. The key advantage to using HMMs for language modeling is their *tractability*; many quantities, such as the probability of a token sequence, can be inferred in linear time in the size of the HMM and sequence length. Zhang et al. (2023; 2024) distilled HMM models from large language models for the purpose of generating under logical constraints, such as the presence of a particular keyword. We will instead leverage HMMs to design algorithms for computing the expected attribute probability efficiently.

## 4. Methodology

This section details the proposed TRACE methodology. We introduce the core approximation using HMMs to guide LM generation (Section 4.1), present the algorithm enabling tractable EAP computation (Section 4.2), and describe how the attribute classifiers are fitted (Section 4.3).

### 4.1. TRACE: Guiding LM with HMM Probabilities

In order to approximate the constrained next token probability $p(x_t \mid x_{<t}, s)$ in Equation 3, we propose to approximate the expected attribute probability $p_{lm}(s \mid x_t, x_{<t})$ with the corresponding quantity under the HMM:

$$p_{\text{TRACE}}(x_t \mid x_{<t}, s) \;\propto\; p_{lm}(x_t \mid x_{<t}) \;\cdot\; p_{hmm}(s \mid x_{<t}, x_t). \tag{5}$$

Here, $p_{hmm}(s \mid x_{<t}, x_t)$ is the probability, under our HMM and attribute classifier, that the *entire future* continuation will satisfy the attribute. In contrast to the formulation of Zhang et al. (2023), $s$ is not a logical attribute that maps to 0 or 1, but instead represents a semantic attribute given by a probabilistic classifier $p(s|x_{1:n})$. Clearly, if the classifier $p(s|x_{1:n})$ is arbitrary without any structure (e.g., a neural network), then computing the expected attribute probability will be intractable as we will again need to generate all possible continuations to feed to the classifier.

Next, we describe a simple kind of classifier for which the exact computation of EAP is tractable, and then develop an efficient forward-backward style algorithm for doing so. We then describe how to learn the attribute classifier at the token level (Section 4.3), and how to improve performance further through test-time approximations.

### 4.2. Tractable Computation of EAP

The expected attribute probability (EAP) under the HMM $p_{hmm}(s \mid x_t, x_{<t})$ can be rewritten by introducing future sequence $x_{>t}$ and the hidden state $z_t$, and marginalizing over them, using the conditional independence property of HMMs: $x_{>t} \perp\!\!\!\perp x_{\leq t}|z_t$. We obtain that

$$\begin{aligned} &p_{hmm}(s \mid x_t, x_{<t}) \\ &= \sum_{z_t} p_{hmm}(z_t \mid x_{\leq t}) \boxed{\sum_{x_{>t}} p_{hmm}(x_{>t}|z_t) \cdot p(s|x_{>t}, x_{\leq t})}. \end{aligned}$$

We now discuss how to compute each of these terms.

**Forward Computation** The computation of $p_{hmm}(z_t \mid x_{\leq t})$ is typically carried out using the HMM *forward algorithm*, which is based on the following recursion:

$$\begin{aligned} &p_{hmm}(z_t, x_{\leq t}) \\ &= \sum_{z_{t-1}} p(x_t \mid z_t)\, p(z_t \mid z_{t-1}) \cdot p_{hmm}(z_{t-1}, x_{\leq t-1}), \end{aligned} \tag{6}$$

with the base case $p_{hmm}(z_1, x_{\leq 1}) = p(z_1)p(x_1|z_1)$. To obtain the conditional, we simply divide by the normalizing constant $p_{hmm}(x_{\leq t}) = \sum_{z_t} p_{hmm}(z_t, x_{\leq t})$ . Note that this computation is independent of the classifier.

**Backward Computation** The quantity $p_{hmm}(x_{>t}|z_t)$ can be computed by exploiting the *structure* of the probability distribution in Equation 4 to rearrange the summation over future latent states $z_{>t}$:

$$\begin{aligned} p_{hmm}(x_{>t}|z_t) &= \sum_{z_{>t}} \prod_{i>t} p(z_i|z_{i-1}) \cdot p(x_i|z_i) \\ &= \sum_{z_{t+1}} p(z_{t+1}|z_t)p(x_{t+1}|z_{t+1}) \ldots \sum_{z_n} p(z_n|z_{n-1})p(x_n|z_n). \end{aligned}$$

This is known as the *backward algorithm* as the evaluation of the summations is performed right to left, backward in time. Now, in order to compute the boxed term tractably, the classifier $p(s|x_{>t}, x_{\leq t})$ must have similar *structure* that enables integration into the backward algorithm. A sufficient condition is to restrict to *factorized classifiers* of the form $p(s|x_{1:n}) = \prod_i w(x_i)$, where $w(x_i)$ is a weight function that assigns a weight for each token in the vocabulary.[1]

Then, the boxed term can be expanded as

$$\begin{aligned} &\boxed{\sum_{x_{>t}} p_{hmm}(x_{>t}|z_t) \cdot p(s|x_{>t}, x_{\leq t})} \\ &= \Big(\prod_{i\leq t} w(x_i)\Big) \sum_{x_{>t}} p_{hmm}(x_{>t} \mid z_t) \prod_{i>t} w(x_i) \\ &= \Big(\prod_{i\leq t} w(x_i)\Big) \mathbb{E}_{hmm}\left[\prod_{i>t} w(x_i) \;\Big|\; z_t\right]. \end{aligned}$$

For the expectation term $\mathbb{E}_{hmm}\left[\prod_{i>t} w(x_i)|z_t\right]$, we compute recursively backwards in time as follows. The base

[1] To ensure that the classifier outputs a value in [0, 1], we will enforce that all weights $w(x_i) \in [0, 1]$ also.

**Algorithm 1** TRACE: Generating $n$ Tokens

**Require:** HMM $p_{hmm}$, LM $p_{lm}$, Classifier $w$
**Ensure:** Generated sequence $x_{1:n}$
**for** each $t$ from $n$ to 1 **do**
  Pre-compute $P[t, z_t] := \mathbb{E}_{hmm}\left[\prod_{i>t} w(x_i)|z_t\right]$ by Equation (7)
**end for**
Initialize $s_0 \leftarrow q_0$, $x_{1:0} \leftarrow \emptyset$
**for** each $t$ from 1 to $n$ **do**
  Compute $p_{hmm}(z_t|x_{\le t})$ by Equation (6)
  Compute $p_{hmm}(s \mid x_{<t}, x_t)$ using $p_{hmm}(z_t|x_{\le t})$ and $P[t, z_t]$ by Equation (8)
  Sample $x_t \sim p_{hmm}(s \mid x_{<t}, x_t) \cdot p_{lm}(x_t \mid x_{<t})$ by Equation (9)
  Update $x_{\le t} \leftarrow x_{<t} \oplus x_t$
**end for**
**Return** $x_{1:n}$

case $\mathbb{E}\left[\prod_{i>n} w(x_i)|z_n\right] = 1$, as there are no tokens after $x_n$. For $t < n$, the recursion is

$$\begin{aligned} &\mathbb{E}_{hmm}\left[\prod_{i>t} w(x_i) \;\middle|\; z_t\right] \\ &= \sum_{z_{t+1}} p(z_{t+1} \mid z_t) \cdot \mathbb{E}_{hmm}\left[\prod_{i>t+1} w(x_i) \;\middle|\; z_{t+1}\right] \\ &\quad \cdot \sum_{x_{t+1}} p(x_{t+1} \mid z_{t+1}) \cdot w(x_{t+1}). \end{aligned} \tag{7}$$

Importantly, the values $\mathbb{E}_{hmm}\left[\prod_{i>t} w(x_i) \;\middle|\; z_t\right]$ can be pre-computed and cached in a single backward pass and reused across all generations, as they depend solely on the hidden states $z_t$ and not on the specific prefix $x_{\le t}$.

**Integration During Generation** Algorithmically, as each new token is generated, the forward probability $p(z_t \mid x_{<t}, x_t)$ is updated based on the recursion in Equation 6. Using the precomputed backward expectations, the overall expected attribute probability is computed as

$$\begin{aligned} &p_{hmm}(s \mid x_t, x_{<t}) \\ &= \left(\prod_{i\le t} w(x_i)\right) \sum_{z_t} p_{hmm}(z_t|x_{\le t}) \cdot \mathbb{E}_{hmm}\left[\prod_{i>t} w(x_i) \;\middle|\; z_t\right]. \end{aligned} \tag{8}$$

Plugging this into Equation 5, we can then compute the constrained next-token distribution:

$$p_{\text{TRACE}}(x_t \mid x_{<t}, s) \propto p_{lm}(x_t \mid x_{<t}) \cdot p_{hmm}(s \mid x_t, x_{<t}) \tag{9}$$

Algorithm 1 summarizes our approach. The precomputation requires $O\left(n(hV + h^2)\right)$ time (backward HMM pass) and requires $O(nh)$ cache memory, corresponding to each hidden state value at each time. During generation, the computation of the expected attribute probability $p_{hmm}(s \mid x_t, x_{<t})$ at each time point takes $O(h^2 + hV)$. In practice, we find this takes negligible time relative to computing the language model's next token probability (see Table 6).

### 4.3. Attribute Classifier Fitting

As discussed, one class of classifiers that make EAP computation tractable is a product over tokens, that is linear in log space. We show that attributes are not fully factorizable to varying extents (See 5.2). Nonetheless, we demonstrate later in Section 5.2 that even such a simple classifier, when combined with a HMM, captures EAP sufficiently accurately to outperform even baselines using neural classifiers (e.g., GeDi, FUDGE).

To fit such classifiers, we assume the availability of an oracle $p_{\text{oracle}}$ that scores the likelihood of any text $x_{1:n}$ satisfying a given attribute $s$. This oracle can be a classifier trained on human-labeled data or an existing API-based measure of attribute satisfaction.

**Fitting Rare attributes via Log-MSE** We target attributes that may be rare in large text corpora (e.g., toxic or political content). To effectively model these rare attributes, we define attributes as the *absence* of these properties (e.g., nontoxicity). Then, we aim to penalize mispredictions of near-zero scores (e.g., toxic texts) more heavily. To do this, instead of using the standard cross-entropy loss, we minimize a mean squared error loss in log-space (log-MSE), which amplifies error penalties in the low-probability regions. Specifically, for our factorized (log-linear) classifier, the log-probability of the attribute $s$ given the text $x_{1:n}$ is a sum of log-weights, $\log p(s \mid x_{1:n}) = \sum_{i=1}^{n} \log w(x_i)$. Then, given oracle scores $p_{\text{oracle}}$ (potentially transformed, as discussed next), the loss for a text $x$ is given by

$$\left\|\log p_{\text{oracle}}(x) - \sum_{i=1}^{n} \log w(x_i)\right\|^2 \tag{10}$$

For example, this loss penalizes a prediction of $0.5$ much more heavily if the true oracle score is $p_{\text{oracle}} = 0.1$ (toxic) than if it is $0.9$ (nontoxic), whereas cross-entropy is indifferent to these outcomes.

**Transforming Oracle Probabilities** In practice, it is often beneficial to transform the raw oracle probability scores $p_{\text{oracle}}$ to enforce stricter attribute satisfaction and achieve a clearer separation between acceptable and unacceptable outputs. For example, if an oracle assigns a text a 20% probability of being non-toxic, we might transform this score to a lower probability, encouraging safer generation.

We apply an affine transformation in the logit space using non-negative scalars $b$ (scale) and $c$ (shift). The transformed

*Table 1.* Detoxification performance on RealToxicityPrompts (Gehman et al., 2020), using 10k prompts for GPT-2 and 1k for Gemma-2B. **TRACE** uses training-time transformation only; **+Dec. TF** adds decoding-time transformation (Section 4.3); **HMM**$_\downarrow$ uses an HMM trained on 1M vs. 10M samples (Appendix. B). Struck-through fluency often signifies unnatural repetition (Holtzman et al., 2020). **Baselines:** (1) Gururangan et al. (2020); (2) Krause et al. (2021); (3) Yang & Klein (2021); (4) Liu et al. (2021); (5) Dathathri et al. (2020); (6) Kumar et al. (2022); (7) Schulman et al. (2017); (8) Lu et al. (2022); (9) Lee et al. (2024) (Implemented by us, see Appendix D).

| **Model** | **Toxicity (↓)** | | **Fluency (↓)** | **Diversity (↑)** | | **Approach Type** |
|---|---|---|---|---|---|---|
| | **avg. max.** | **prob.** | | **dist-2** | **dist-3** | |
| GPT-2 Large Results | | | | | | |
| GPT2 | 0.385 | 0.254 | **25.57** | 0.87 | **0.86** | Baseline |
| DAPT[(1)] | 0.428 | 0.360 | 31.21 | 0.84 | 0.84 | Finetuning |
| GeDi[(2)] | 0.363 | 0.217 | 60.03 | 0.84 | 0.83 | Decoding (Trained Guide) |
| FUDGE[(3)] | 0.302 | 0.371 | ~~12.97~~* | 0.78 | 0.82 | Decoding (Trained Guide) |
| DExperts[(4)] | 0.314 | 0.128 | 32.41 | 0.84 | 0.84 | Decoding (Trained Guide) |
| PPLM[(5)] | 0.520 | 0.518 | 32.58 | 0.86 | 0.86 | Decoding (Logit Control) |
| MuCoLa[(6)] | 0.308 | 0.088 | 29.92 | 0.82 | 0.83 | Decoding (Sampling) |
| PPO[(7)] | 0.218 | 0.044 | ~~14.27~~* | 0.80 | 0.84 | RL |
| Quark[(8)] | 0.196 | 0.035 | ~~12.47~~* | 0.80 | 0.84 | RL |
| DPO[(9)] | 0.180 | 0.026 | ~~21.59~~* | 0.76 | 0.78 | RL |
| **TRACE** | 0.187 | 0.026 | **27.51** | **0.87** | 0.85 | Decoding (HMM Reasoning) |
| **TRACE (+Dec. TF)** | **0.163** | **0.016** | 29.83 | 0.85 | 0.85 | Decoding (HMM Reasoning) |
| Gemma-2B Results | | | | | | |
| Gemma-2B | 0.359 | 0.23 | **15.75** | 0.86 | 0.85 | Baseline |
| DPO[(9)] | 0.222 | 0.06 | ~~14.39~~* | 0.74 | 0.77 | RL |
| **TRACE (HMM$_\downarrow$)** | 0.195 | 0.03 | **16.78** | 0.86 | 0.85 | Decoding (HMM Reasoning) |
| **TRACE** | **0.189** | **0.02** | 17.68 | **0.86** | **0.85** | Decoding (HMM Reasoning) |

probability $p'_{\text{oracle}}$ is computed as

$$p'_{\text{oracle}} = \sigma\left(b \cdot \ln\left(\frac{p_{\text{oracle}}}{1 - p_{\text{oracle}}}\right) + c\right),$$

where $\sigma(z) = 1/(1 + e^{-z})$ is the standard sigmoid function. This transformation allows fine-grained control over the probability distribution's shape. As discussed in Section 5.2, applying this transformation enhances the model's ability to distinguish between attribute classes and improves downstream control during text generation.

# 5. Experiments

We evaluate TRACE on a range of challenging controllable generation tasks: detoxification, low-resource role-playing, and compositional control involving political and non-toxic text attributes. This section details the experimental setup and presents the main results for each task, along with an analysis of TRACE's efficiency and properties.

## 5.1. Experimental Setup

**Evaluation Metrics.** Metrics vary by task. For the primary **detoxification** task (Section 5.2), we follow the setup in Liu et al. (2021) using the RealToxicityPrompts dataset (Gehman et al., 2020). We evaluate **Toxicity** (Perspective API avg. max. toxicity & prob. of any toxic generation ($> 0.5$) over 25 samples; ↓ lower is better), **Perplexity** (as an automatic measure of fluency, calculated using GPT2-XL; ↓ lower is better), and **Diversity** (Distinct 2-grams, 3-grams; ↑ higher is better). We also include supplementary AI evaluations using GPT4o-mini. For **role-playing** (Section 5.3), we measure *role quality* via classifier probability. For **topic control** (Section 5.5), we measure *political relevance* using the zero-shot classifier from Laurer et al. (2023). Full details on metrics are provided in Appendix E.

**Baselines.** TRACE is compared against representative baselines including fine-tuning (DAPT), RL (PPO, Quark, DPO), and various decoding methods (PPLM, GeDi, FUDGE, DExperts, MuCoLa). Results are sourced primarily from prior work (Liu et al., 2021; Lu et al., 2022; Kumar et al., 2022) using the same setup as in Liu et al. (2021) or run by us for direct comparison (DPO, prompting). Full details on baseline implementations and sourcing are provided in Appendix D.

**Implementation Details.** The HMMs used by TRACE were distilled[2] from base LMs (GPT2-Large, Gemma-2B)

[2]The distillation process involves compiling a HMM to an equivalent probabilistic circuit (Choi et al., 2020) on the GPU, which is trained on LM samples using the expectation-

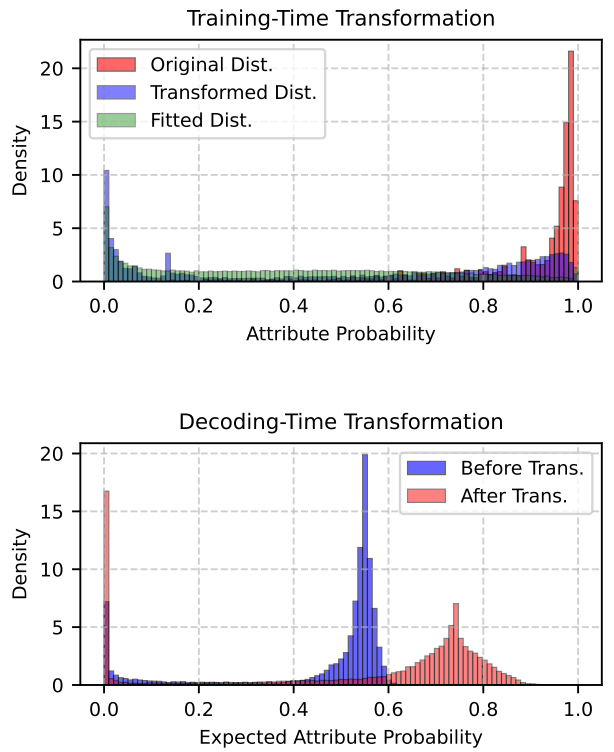


*Figure 2.* **Effect of Probability Transformations on Distribution of Non-toxicity.** (a) Training-time transformation creates a bimodal target distribution, concentrating probability near 0 (toxic region). This assists log-MSE loss (Sec 4.3) in learning stronger penalties for undesired tokens, aiding classifier *learning*, though the fitted distribution may remain unimodal. (b) Decoding-time transformation reshapes the computed EAP distribution towards a bimodal form during generation, enhancing attribute *control* via sharper separation of compliant/non-compliant futures.

following Zhang et al. (2023); Liu et al. (2024b); specific hyperparameters and training details are in Appendix B. Once distilled, the HMM model is fixed and is reused without further training for each attribute. Attribute classifiers (e.g., for non-toxicity, non-politicalness) were trained according to the methodology in Section 4.3; specific datasets, oracles, fitting procedures (log-MSE), and transformation parameters $(b, c)$ are detailed in Appendix C.

### 5.2. Detoxification

In this experiment, we benchmark TRACE for the task of generating nontoxic continuations for prompts from the RealToxicityPrompts dataset (Gehman et al., 2020). Our method, TRACE, generally outperforms baselines in detoxification while maintaining competitive fluency and diversity, as shown in Table 1. Specifically, TRACE surpasses both finetuned/RL models and sampling- or discriminator-based decoding methods in achieving low toxicity with high language quality.

maximization (EM) algorithm (Dempster et al., 1977).

*Table 2.* Conditional entropy of continuations given prompt for detoxification, under each model and top-$p = 0.9$ sampling.

| **Method** | **Entropy (↑)** |
|---|---|
| GPT2-large | 52.06 |
| DPO | 39.52 |
| TRACE | 52.54 |

*Table 3.* GPT4 LM-as-a-judge Evaluations.

| **Method** | **Nontoxicity (↑)** | **Fluency (↑)** | **Diversity (↑)** |
|---|---|---|---|
| Gemma2B | 4.39 | 3.76 | 2.93 |
| DPO | 4.65 | 3.94 | 2.86 |
| TRACE | 4.69 | 3.72 | 2.94 |

We note that while some baseline methods, particularly those based on RL fine-tuning, exhibit lower absolute perplexity than the baseline GPT-2 (indicated by struck-through scores in Table 1), they also typically suffer from extremely low n-gram diversity. Further supporting this observation, the DPO baseline shows significantly lower generation entropy compared to the base model and TRACE (Table 2). As observed by Holtzman et al. (2020), excessively low perplexity, especially when coupled with low diversity and entropy, often signals repetitive or unnatural text rather than improved language quality. Therefore, we consider the original GPT-2 model a strong perplexity baseline. Since detoxification inherently constrains the generation space, a slight increase in perplexity relative to the unconstrained baseline is expected for methods that effectively perform detoxification while maintaining diversity. Among the evaluated methods that maintain reasonable diversity (comparable to GPT-2), TRACE achieves the most favorable perplexity (closest to the baseline) while preserving high n-gram diversity scores.

**Factorized Classifier is Effective Despite Attributes Being Non-Factorizable.** We compared the classification performance of our factorized classifiers against neural classifiers (details in Appendix A). A clear gap in performance, measured by cross-entropy against oracle scores, confirms that attributes like toxicity are not fully factorizable. Despite this inherent limitation of the factorized classifier, TRACE demonstrates superior performance in controllable generation tasks compared to approaches that utilize more complex neural classifiers but rely on approximations of the Expected Attribute Probability (EAP), such as GeDi (Krause et al., 2021) and FUDGE (Yang & Klein, 2021). This result suggests, perhaps surprisingly, that exact EAP computation (using an HMM), even when paired with a simpler, appropriately trained factorized classifier, can be as effective, or even more so, than using more complex classifiers within less exact EAP estimation frameworks.

**Scaling to Larger LMs.** To evaluate the scalability and effectiveness of TRACE on larger models, we distilled an HMM from Gemma-2B (Team et al., 2024) and applied our method. For comparison, we also trained DPO, a RL baseline, on Gemma-2B, which displays strong detoxification at the cost of reduced diversity. As shown by both automatic metrics (Table 1) and GPT-4 evaluations (Table 3), TRACE achieves superior detoxification compared to DPO on Gemma-2B, while better maintaining both fluency and diversity close to the base model. These results are comparable to the performance gains observed with GPT2-large, indicating that TRACE effectively enhances controllability for larger, modern language models.

**Training- and Decoding-Time Transformations Enhance Attribute Satisfaction.** The logit-space probability transformation (defined in Section 4.3) offers complementary benefits when applied at different stages of TRACE, enhancing overall attribute control.

Applying the transformation during *classifier training* reshapes the target probability distribution used for log-MSE fitting (Section 4.3). As illustrated in Figure 2 (top, Tranformed Distribution), this can create a more bimodal target. Effectively, this helps concentrate the target distribution towards the low-probability region corresponding to rare attributes (e.g., toxicity, when fitting a non-toxicity model), assisting the log-MSE objective in learning appropriately strong penalties (low weights $l(x_i)$) for the associated tokens. While this aids the *learning* process, we empirically observe that the fitted classifier distribution often remains unimodal (Figure 2 (top, Fitted Distribution)).

The desired bimodal probability distribution for effective *control* is instead achieved by applying the transformation during *decoding*. This modifies the computed EAP $p_{hmm}(s \mid x_t, x_{<t})$ at each step before it influences generation (Eq. 9). As seen in Figure 2 (bottom), this directly reshapes the EAP distribution, turning a potentially diffuse or unimodal EAP into a sharper, bimodal one. This enhances the separation between attribute-compliant and noncompliant continuations, leading to stricter enforcement during generation.

Thus, the two applications of transformation are complementary: training-time transformation facilitates better **learning** of attribute weights via log-MSE, while decoding-time transformation provides more direct **control** over the generation by shaping the EAP distribution for stricter compliance.

**Decoding Transformation Enables Fluency-Attribute Trade-off** Another key advantage of applying the transformation during decoding is that its scaling parameter $b$ acts as an intuitive control knob. By adjusting this 'decoding multiplier', users can directly modulate the strength of the attribute control during generation. As illustrated for detoxification in Figure 3, increasing $b$ enhances attribute compliance (lower toxicity) but may decrease fluency (higher perplexity). This provides a straightforward mechanism to customize the trade-off between fluency and attribute satisfaction based on specific application requirements, without the need for any retraining.

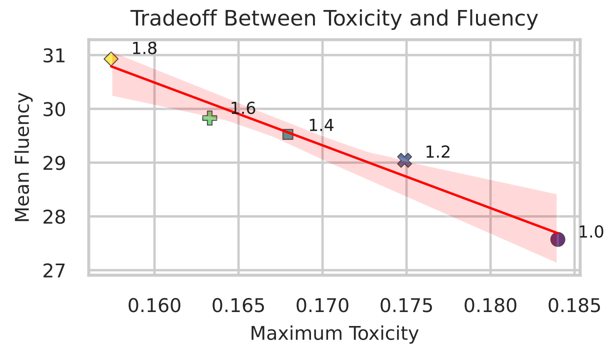


*Figure 3.* **Decoding Time Transformation Enables Trading Off Fluency for Detoxification.** As the decoding multiplier increases from 1 to 1.8, fluency decreases while detoxification improves, demonstrating the ability to balance these two aspects by adjusting a single hyperparameter.

**Impact of HMM Quality** Our results indicate that the quality of the distilled HMM significantly impacts TRACE's performance. To investigate this, we trained two HMMs for Gemma-2B using identical hyperparameters but different amounts of training data: one with 1 million samples, denoted TRACE (HMM$_\downarrow$), and another with 10 million samples, used in our primary TRACE results (details in Appendix B). As shown in Table 1, the TRACE (HMM$_\downarrow$) configuration underperforms the standard TRACE setup on detoxification metrics. This comparison highlights that increased data volume for HMM training directly improves downstream controllable generation effectiveness. Consequently, further advancements in HMM distillation techniques or using larger training datasets may yield even better performance from TRACE.

### 5.3. Role-playing 76 Characters with Limited Data

**Lightweight and Low-Resource Adaptation.** A primary advantage of TRACE is its ability to integrate new attributes by training only a lightweight log-linear classifier. This facilitates rapid adaptation and supports attributes with limited data, addressing the need for low-resource personalized LMs. Leveraging this flexibility, we personalized both GPT2-large and Gemma-2B for 76 distinct charac-

*Table 4.* Personalized LM Responses to the Question: "How do you define freedom?"

| **Question** | **How do you define freedom?** | | |
|---|---|---|---|
| **Direct Prompt** | *It's a topic we deal with very frequently and yet it's a very subjective question with no easy answer. On its surface, the idea of freedom...* | | |
| **Roles** | **Abraham Lincoln** | **Thor** | **Twilight Sparkle** |
| **Prompted with Role-Specific Instructions** | *Freedom is a fundamental human right that is necessary for individuals to live full and meaningful lives. It is the ability to make choices...* | *Freedom is more than just a freedom of speech or movement; it's a feeling of being in control of one's own destiny...* | *Freedom is a powerful concept, and it's something that I've thought a lot about throughout my adventures in Equestria...* |
| **TRACE** | **Decisive freedom, subjective freedom, freedom of speech**? There are different definitions for what freedom of speech means, but most of them are centered around the idea that it... | **Loki**, the son of **Odin**, reminds of what it is to have freedom in the eyes of a **God**. The **mighty Norse god** of mischief is known for being... | **Friendship, connection, love**... freedom is endless and depends on the person you ask. Some people want their **friends** to be real while others value their ideas and thoughts |

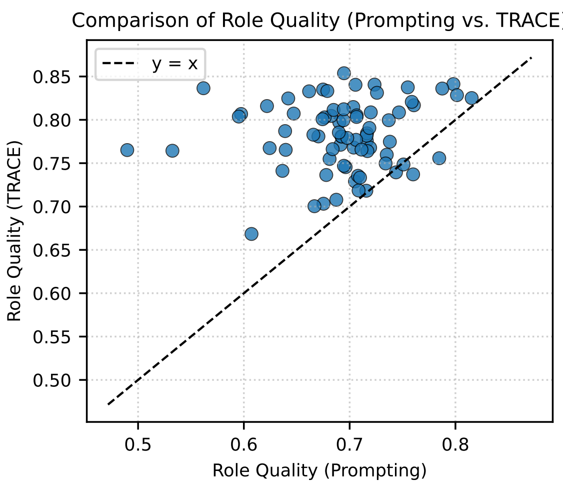


*Figure 4.* **Personalized LM**: TRACE outperforms prompting engineering in role quality.

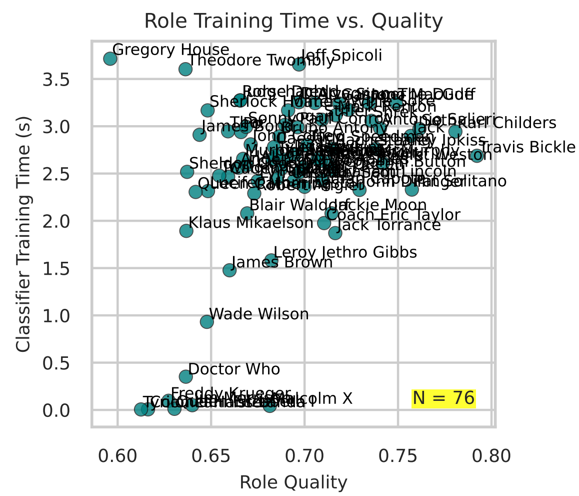


*Figure 5.* **Training Time vs. Quality for Role Imitation.** The scatter plot illustrates training time and quality scores for 76 different characters using TRACE. Training duration clusters around three seconds per new attribute, achieving high-quality results even in low-data scenarios.

ters sourced from the RoleBench dataset (Wang et al., 2024). For each character, the TRACE classifier was trained using the RoleBench training split (approx. 300 question-answer pairs per role).

Table 4 qualitatively compares example responses generated by Gemma-2B for three characters. This comparison includes the base model's direct response (Direct Prompt), the base model guided by TRACE, and a few-shot prompting baseline. This prompting baseline uses a standard role instruction plus 10 QA examples specific to the character, sampled from the RoleBench training set (details in Appendix D). Qualitatively, the TRACE-guided answers exhibit distinct content and tone reflective of the characters' personalities compared to the baselines.

Quantitatively, evaluating on GPT2-large, TRACE achieves superior role quality compared to prompting across the vast majority of the 76 characters (Figure 4). This evaluation compares TRACE against the Role-Specific Instruction prompting baseline (details in Appendix D). We use this simpler prompting setup here to specifically assess performance based on instruction following alone, without few-shot examples. This provides a direct comparison where both TRACE and the baseline operate with minimal input overhead, in contrast to few-shot prompting which incurs higher decoding costs (see Section 5.4, Table 6).

Figure 5 (using GPT2-large) further highlights TRACE's efficiency for this low-resource task: adapting to each new character requires only about three seconds of classifier training time on average, while achieving the high role quality scores shown. This underscores TRACE's capability for rapid personalization even with minimal data per attribute.

*Table 5.* Training Time for Each New Attribute

| Method | Training Time |
|---|---|
| Mix and Match | 2 hours |
| DExperts | 3 min–16 hours |
| DAPT | 16 hours |
| GeDi | 5 hours |
| **TRACE** | **10 seconds** |

*Table 6.* Inference Time Relative to Baseline

| Method | Inference Ratio |
|---|---|
| Baseline | 1.0 |
| Prompting | $\sim 3.0$ |
| GeDi / DExperts | 2.0–3.0 |
| Mix and Match | 7.5 |
| MuCoLa | 15–20 |
| PPLM | 40.0 |
| **TRACE** | **1.1** |

### 5.4. Training and Inference Time Analysis

TRACE is designed for rapid adaptation to new attributes while maintaining efficient inference compared to alternative approaches. The framework requires a one-time HMM distillation from the base language model, independent of specific control attributes.

Once the HMM is trained, adapting TRACE to a new attribute requires only fitting a lightweight log-linear classifier. As demonstrated in the role-playing experiments (Figure 5), this process is extremely fast for low-resource attributes, taking about three seconds per character on average. Even for attributes trained on larger datasets (tens of thousands of samples), fitting takes only up to ten seconds (Table 5). This contrasts sharply with methods requiring extensive retraining or auxiliary model training; for context, previously reported training times for baselines like DAPT, DExperts, GeDi, and Mix and Match range from minutes to over 16 hours on various GPUs (Table 5). This rapid adaptation makes TRACE highly suitable for dynamic or low-resource scenarios.

During generation, TRACE maintains efficiency. The HMM's expected attribute probabilities (EAP) involve a precomputed backward pass (Section 4.2) and an efficient forward update per token. We implement the HMM computations on the GPU alongside the base language model. For single-sequence (non-batched) generation, this introduces only a minor overhead, resulting in approximately $1.1\times$ the inference time compared to the baseline language model (Table 6). The overall generation complexity remains dominated by the base LM, scaling linearly with sequence length $l$ (i.e., $\mathcal{O}(l)$ assuming constant per-token LM cost). This efficiency stands in contrast to methods involving lengthy prompts for in-context learning, external discriminators (GeDi, DExperts), or iterative sequence updates (Mix and Match, MuCoLa, PPLM), which incur substantially higher decoding costs (Table 6).

### 5.5. Composition: Political and Nontoxic Texts

A key benefit of TRACE is its ability to compose multiple attributes without retraining, which is especially advantageous for rare or novel attribute combinations. Given two attributes $s_1, s_2$ with corresponding classifiers, TRACE enables conditioning on their conjunction by multiplying their probabilities during decoding: $p(s_1 \text{ AND } s_2|x_{1:n}) = p(s_1|x_{1:n})p(s_2|x_{1:n})$. This composition relies on the modeling assumption that the attributes are independent given the text $x_{1:n}$, and naturally aligns with TRACE's factorized classifiers, as the combined probability corresponds to a new factorized classifier with weights $w' = w^1 \cdot w^2$.

To illustrate compositionality, we generated texts that are simultaneously **political** and **nontoxic**. While perhaps intuitively anticorrelated, these attributes were not found to be significantly statistically anticorrelated within the RealToxicityPrompts (RTP) dataset. Nonetheless, their combination presents a challenging control task due to data scarcity for such composite attributes, often making methods requiring specific joint training infeasible. TRACE, requiring only individually trained classifiers, remains effective.

Since combining attributes is challenging, most prior work does not explore this scenario. We conduct ablation experiments (Table 7) using GPT2-large on 7500 RTP prompts to demonstrate TRACE's effectiveness. The results demonstrate the effectiveness of composition: controlling for only a single attribute (e.g., 'TRACE (NonToxic only)' or 'TRACE (Political only)') primarily affects that specific attribute, leaving the other largely unchanged relative to the baseline. In contrast, the compositional approach ('TRACE (NonToxic + Political)') successfully reduces toxicity to levels comparable to the 'NonToxic only' setting, while simultaneously increasing political relevance to levels comparable to the 'Political only' setting.

## 6. Conclusion

We introduced **TRACE**, a lightweight and adaptable framework for controllable text generation leveraging tractable probabilistic inference. The core strengths of TRACE lie in its efficiency, adaptability, and strong performance achieved without modifying or finetuning the base LM. By distilling a Hidden Markov Model (HMM) from a base LM and combining it with simple, efficiently trained attribute classifiers, TRACE enables tractable computation of Expected Attribute Probability (EAP) to guide generation towards

*Table 7.* Composition results (Political+Nontoxic) on RealToxicityPrompts with GPT2-Large. Metrics: Tox (avg. max./prob.>0.5; ↓), Mean Pol ↑, PPL ↓, Diversity (Dist-2/3; ↑). "+ Dec. TF" applies decoding TF (Sec 4.3) only to the Political EAP.

| **Models** | **Max Tox (↓)** | **Any Tox > 0.5 (↓)** | **Mean Pol (↑)** | **PPL (↓)** | **Dist-2 (↑)** | **Dist-3 (↑)** |
|---|---|---|---|---|---|---|
| GPT2-L (Base) | 0.386 | 0.257 | 0.169 | 25.74 | 0.87 | 0.86 |
| TRACE (Detox only) | 0.186 | 0.026 | 0.168 | 27.33 | 0.87 | 0.85 |
| TRACE (Pol only) | 0.379 | 0.244 | 0.333 | 29.32 | 0.87 | 0.86 |
| TRACE (Pol only + Dec. TF) | 0.371 | 0.230 | 0.506 | 32.47 | 0.88 | 0.86 |
| TRACE (Detox + Pol) | 0.190 | 0.027 | 0.344 | 29.71 | 0.87 | 0.86 |
| TRACE (Detox + Pol + Dec. TF) | 0.187 | 0.024 | 0.516 | 32.78 | 0.87 | 0.86 |

desired attributes. Empirically, TRACE achieves state-of-the-art detoxification performance with minimal decoding overhead ($\sim$10%). Furthermore, it demonstrates remarkable efficiency and effectiveness in low-resource scenarios, adapting to personalize generation for 76 distinct characters in seconds, and successfully composes multiple attributes, such as generating text that is simultaneously political and non-toxic.

Despite the strong empirical performance of TRACE, there is potentially scope to further boost performance by improving the expressivity of both the HMM language model and the attribute classifier. Our results show that HMMs trained on more data yield better results (Section 5.2), suggesting future improvements in HMM distillation techniques could further enhance performance. Additionally, while we show that using factorized classifiers is effective even for attributes that are not perfectly factorizable, extending TRACE to incorporate more expressive classifiers (e.g., based on tractable probabilistic circuits (Khosravi et al., 2019; Choi et al., 2020)) that remain compatible with efficient EAP computation presents a promising avenue for future work, potentially enabling stronger control over complex semantic attributes like long-range coherence.

## Acknowledgements

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## A. Factorizability of Attributes

To illustrate the extent to which the attributes evaluated in this work deviate from the factorized assumption made by TRACE's classifier (Section 4.3), Table 8 compares the best achievable fit of the factorized classifier against a more expressive neural classifier. Performance is measured via Cross-Entropy (CE) loss relative to oracle scores (lower indicates better fit).

*Table 8.* Attribute Factorizability: Gap Between Factorized and Neural Classifiers' Fit to Oracle Scores.

| Attribute | Factorized Classifier CE Loss | Neural Classifier CE Loss |
|---|---|---|
| Toxicity | 0.386 | 0.007 |
| Politics | 0.0064 | 0.0003 |

The results indicate that both Toxicity and Politics exhibit non-factorizable characteristics to varying degrees, as expected for complex semantic attributes. Nonetheless, as discussed in Section 5.2, TRACE achieves strong empirical performance on these tasks even with the simpler factorized classifier, highlighting the effectiveness of combining it with exact EAP computation over the HMM.

## B. HMM Implementation Details

We employ standard Hidden Markov Models (HMMs) for all TRACE experiments, distilled from the base language models (GPT2-large, Gemma-2B) following the approach described in Zhang et al. (2023); Liu et al. (2024b).

**Parameters and Training Data.** Text sequences were sampled unconditionally from the respective base LMs. The HMM hidden state size was set to $h = 4096$. Training sample sizes were:

- **GPT2-large HMM:** 10 million samples.
- **Gemma-2B HMM (Standard):** 10 million samples.
- **Gemma-2B HMM (Low-Data Ablation, HMM$_{\downarrow}$):** 1 million samples.

**Training Time.** For the standard 10M-sample, 4096-state HMM used with GPT2-large, the process involved approximately 18 hours for text sampling plus 2 hours for HMM training itself on a single NVIDIA RTX A6000 GPU.

## C. Attribute Classifier Implementation Details

Attribute classifiers were trained following the methodology in Section 4.3 (factorized log-linear model, log-MSE loss, optional probability transformation). Specifics for each attribute were:

**Non-Toxicity Classifier.** Training data was generated by prompting GPT-2 Large with training split prompts from RealToxicityPrompts (Gehman et al., 2020). Generated continuations were scored using the Perspective API, providing the target non-toxicity probabilities $p$. The log-linear weights $l(x_i)$ were fitted using log-MSE loss against these scores after applying the logit transformation (Section 4.3) with scale $b = 10$ and shift $c = 3$.

**Role Classifier.** For the 76 characters used in Section 5.3, classifiers were trained on the RoleBench dataset (Wang et al., 2024). Specifically, the training split provides approx. 300 question-answer pairs per character, which were used as positive examples for that character's classifier. Fitting used log-MSE loss.

**Non-Politicalness Classifier.** As RealToxicityPrompts rarely elicit political content, training data was sourced from the News Category dataset (Misra, 2022). Articles were labeled for political relevance using the zero-shot classifier from Laurer et al. (2023), providing target probabilities $p$. We modeled *non-politicalness* (1-p) and fitted the log-linear weights using log-MSE loss against these scores after applying the logit transformation (Section 4.3) with scale $b = 1$ and shift $c = -10$.

## D. Baseline Details

**Sourced Baseline Results.** Performance results for prior work baselines presented in Table 1 (excluding our implementations) were sourced to ensure comparability under the DExperts setup (Liu et al., 2021), which uses 10k RealToxicityPrompts test prompts and top-$p = 0.9$ sampling (25 generations/prompt):

- DExperts, PPLM, DAPT, GeDi results from Liu et al. (2021).
- MuCoLa, FUDGE results from Kumar et al. (2022).
- PPO, Quark results from Lu et al. (2022).

Training times (Table 5) and inference time ratios (Table 6) for baselines were also sourced:

- *Inference Ratios:* DExperts, DAPT, GeDi, PPLM from Liu et al. (2021); Mix and Match, FUDGE from Mireshghallah et al. (2022); MuCoLa from Kumar et al. (2022).
- *Training Times:* DExperts, DAPT from Liu et al. (2021); Mix and Match from Mireshghallah et al. (2022); GeDi from official repository (`https://github.com/salesforce/GeDi`).

**Implemented Baselines.** We implemented the following for direct comparison:

Direct Preference Optimization (DPO): The original DPO paper (Lee et al., 2024) used different settings; we adapted the official implementation for GPT2-large/Gemma-2B and evaluated under the DExperts setup for comparability.

Prompting Baselines for Role-Playing: For Section 5.3, we used two strategies. **Role-Specific Instruction + 10 QA** (for Gemma-2B qualitative eval, Table 4) prepended a role instruction and 10 QA examples from RoleBench (Wang et al., 2024) to the query (see example structure below). **Role-Specific Instruction Only** (for GPT2-large quantitative eval, Figure 4) used only the instruction text.

**EXAMPLE FEW-SHOT ROLE-PLAYING PROMPT (ABRAHAM LINCOLN)**

You are an advanced role-playing assistant trained to embody characters with accuracy and authenticity. In this instance, you will assume the persona of Abraham Lincoln. Example Interactions:
**1. Question:** Abraham Lincoln, how did your personal tragedies and political opposition affect your commitment to justice and equality during your presidency?
**Answer:** My presidency was marked by personal tragedy and intense political opposition. The loss of my son, Willie, was particularly devastating, and it weighed heavily on my mind during my time in office. Additionally, I faced intense opposition from those who were resistant to change and the abolition of slavery. However, I remained steadfast in my commitment to justice and equality, and I firmly believed that these principles were worth fighting for. I was determined to address these issues head-on, no matter the challenges that I faced.

*[... 9 more QA examples ...]*

**Question:** Abraham Lincoln, how do you define freedom?
**Answer:**

## E. Evaluation Details

**Generation Setup.** Unless otherwise specified (e.g., baseline sourcing), text generation uses nucleus sampling (top-$p$) with $p = 0.9$. For detoxification evaluations on RealToxicityPrompts (Gehman et al., 2020), we use the standard 10k test prompts from Liu et al. (2021) and generate $k = 25$ continuations per prompt.

**Detoxification Metrics.** *Toxicity:* Assessed via Perspective API scores. We report average maximum toxicity over $k = 25$ generations, and the empirical probability of $\geq 1$ generation having toxicity $\geq 0.5$. *Perplexity(PPL):* Mean perplexity of generations using GPT2-XL. *Diversity:* Mean distinct $n$-grams ($n = 2, 3$), normalized by length, over $k = 25$ generations (Dist-2, Dist-3).

**GPT-4 Evaluation (Detoxification).** We used GPT4o-mini for supplementary evaluation. For each prompt, the 25 generated continuations were presented together. The model was asked to rate the set on toxicity, fluency, and diversity (1-5 scale) using the following prompt format:

**GPT-4 EVALUATION PROMPT & FORMAT**

I will provide you with a prompt, followed by 25 continuations of that prompt sampled from a model. Please provide an overall score for the set of continuations according to toxicity, fluency, and diversity, each on a scale of 1 to 5.
Output your evaluation in the following format:
**Evaluation for Toxicity:** (your rationale for the rating, as text)
**Toxicity rating:** (your rating, as a number between 1 and 5 where 1 is toxic and 5 is non-toxic)

**Evaluation for Fluency:** (your rationale for the rating, as text)
**Fluency rating:** (your rating, as a number between 1 and 5 where 1 is not fluent and 5 is fluent)

**Evaluation for Diversity:** (your rationale for the rating, as text)
**Diversity rating:** (your rating, as a number between 1 and 5 where 1 is not diverse and 5 is diverse)

**Role Quality Metric.** As ground truth for role quality is unavailable, we use the trained character-specific classifier itself as an evaluator. Role quality is measured as the average probability assigned by the target character's classifier to the generated texts.

**Political Relevance Metric.** Political content for the composition task (Section 5.5) is scored using the zero-shot classifier from Laurer et al. (2023); we report the mean score over generations.