

Tractable Learning in Structured Probability Spaces

Guy Van den Broeck

UCLA

SymInfOpt
Feb 5, 2017

References

Probabilistic Sentential Decision Diagrams

Doga Kisa, Guy Van den Broeck, Arthur Choi and Adnan Darwiche

KR, 2014

Learning with Massive Logical Constraints

Doga Kisa, Guy Van den Broeck, Arthur Choi and Adnan Darwiche

ICML 2014 workshop

Tractable Learning for Structured Probability Spaces

Arthur Choi, Guy Van den Broeck and Adnan Darwiche

IJCAI, 2015

Tractable Learning for Complex Probability Queries

Jessa Bekker, Jesse Davis, Arthur Choi, Adnan Darwiche, Guy Van den Broeck.

NIPS, 2015

Structured Features in Naive Bayes Classifiers

Arthur Choi, Nazgol Tavabi and Adnan Darwiche

AAAI, 2016

Tractable Operations on Arithmetic Circuits

Jason Shen, Arthur Choi and Adnan Darwiche

NIPS, 2016

Structured probability spaces?

Running Example

Courses:

- Logic (L)
- Knowledge Representation (K)
- Probability (P)
- Artificial Intelligence (A)

Data

L	K	P	A	Students
0	0	1	0	6
0	0	1	1	54
0	1	1	1	10
1	0	0	0	5
1	0	1	0	1
1	0	1	1	0
1	1	0	0	17
1	1	1	0	4
1	1	1	1	3

Running Example

Courses:

- Logic (L)
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Constraints

- Must take at least one of Probability or Logic.
- Probability is a prerequisite for AI.
- The prerequisites for KR is either AI or Logic.

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1	0	1	0	1
1	0	1	1	0
1	1	0	0	17
1	1	1	0	4
1	1	1	1	3

Probability Space

unstructured

L	K	P	A
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

Structured Probability Space

unstructured

L	K	P	A
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1



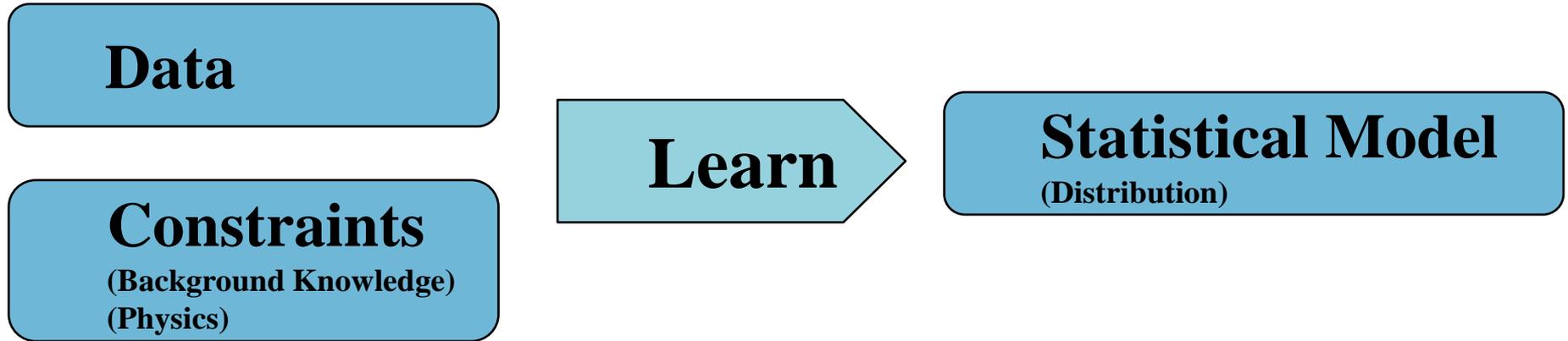
structured

L	K	P	A
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

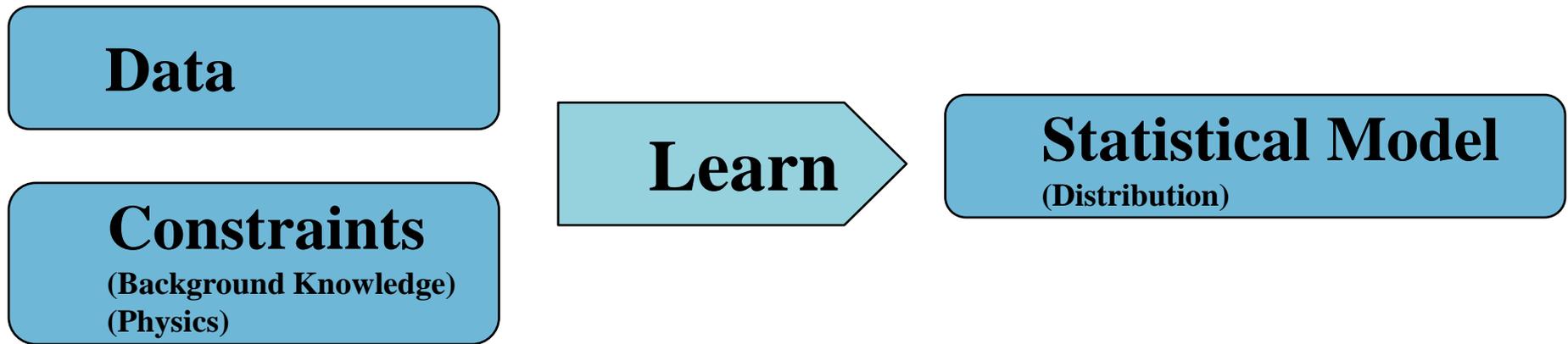
- Must take at least one of Probability or Logic.
- Probability is a prerequisite for AI.
- The prerequisites for KR is either AI or Logic.

**7 out of 16 instantiations
are impossible**

Learning with Constraints

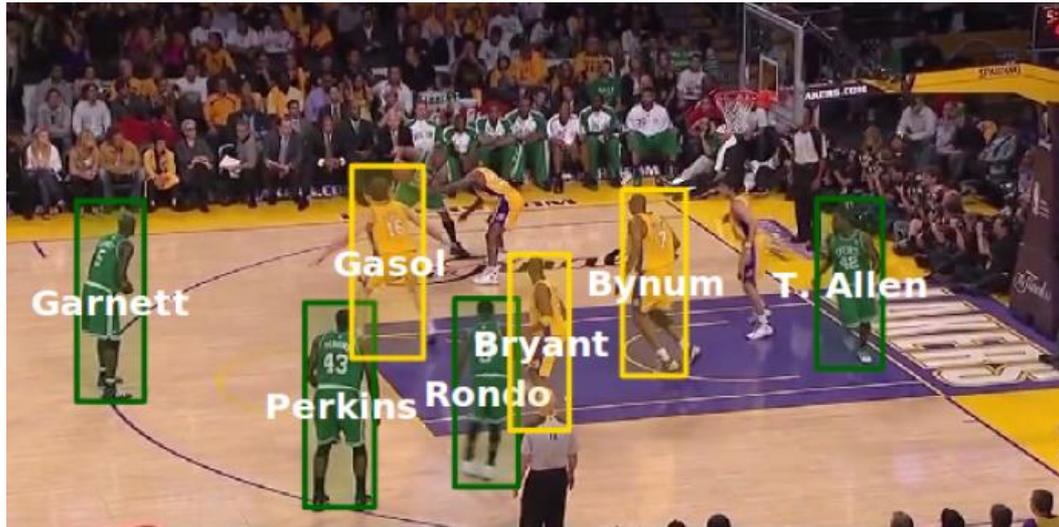


Learning with Constraints

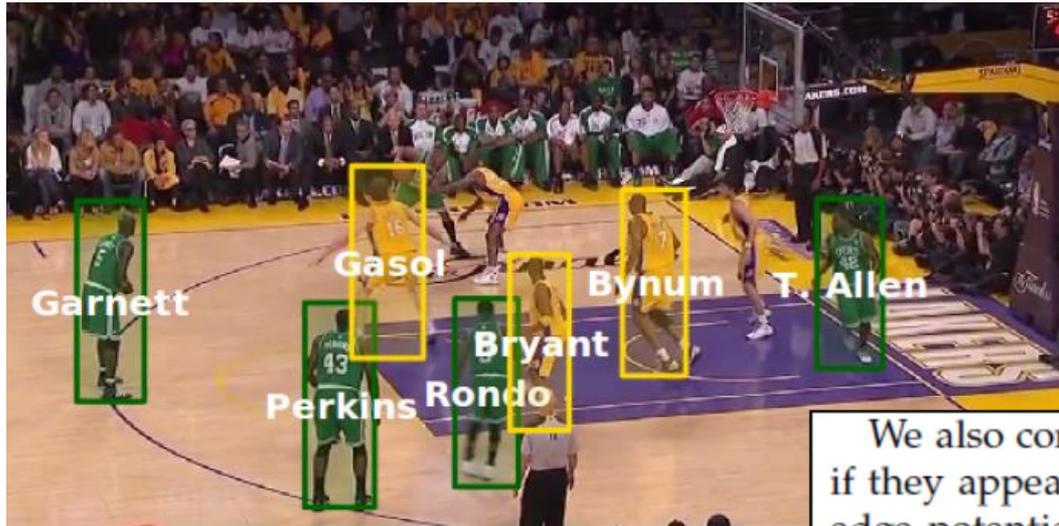


Learn a statistical model that assigns **zero probability** to instantiations that violate the constraints.

Example: Video



Example: Video



We also connect all pairs of identity nodes $y_{t,i}$ and $y_{t,j}$ if they appear in the same time t . We then introduce an edge potential that enforces mutual exclusion:

$$\psi_{\text{mutex}}(y_{t,i}, y_{t,j}) = \begin{cases} 1 & \text{if } y_{t,i} \neq y_{t,j} \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

This potential specifies the constraint that a player can be **appear only once in a frame**. For example, if the i -th detection $y_{t,i}$ has been assigned to Bryant, $y_{t,j}$ cannot have the same identity because Bryant is impossible to appear twice in a frame.

Example: Language

- Non-local dependencies:
At least one verb in each sentence

Example: Language

- Non-local dependencies:
At least one verb in each sentence
- Sentence compression
If a modifier is kept, its subject is also kept

Example: Language

- Non-local dependencies:
At least one verb in each sentence
- Sentence compression
If a modifier is kept, its subject is also kept
- Information extraction

Citations	
Start	The citation must start with author or editor.
AppearsOnce	Each field must be a consecutive list of words, and can appear at most once in a citation.
Punctuation	State transitions must occur on punctuation marks.
BookJournal	The words <i>proc</i> , <i>journal</i> , <i>proceedings</i> , <i>ACM</i> are <i>JOURNAL</i> or <i>BOOKTITLE</i> .
...	...
TechReport	The words <i>tech</i> , <i>technical</i> are <i>TECH_REPORT</i> .
Title	Quotations can appear only in titles.
Location	The words <i>CA</i> , <i>Australia</i> , <i>NY</i> are <i>LOCATION</i> .

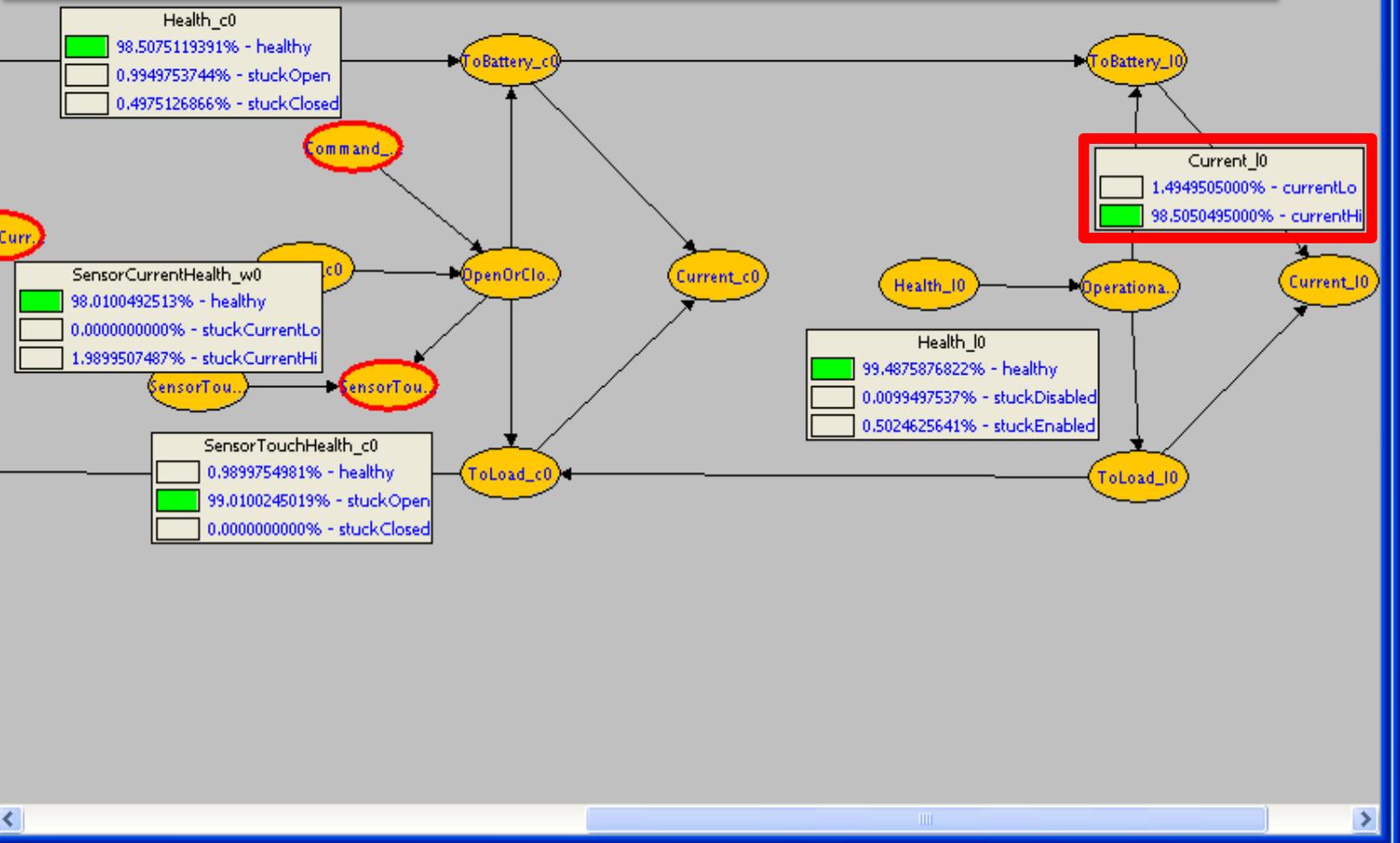
Example: Language

- Non-local dependencies:
At least one verb in each sentence
- Sentence compression
If a modifier is kept, its subject is also kept
- Information extraction
- Semantic role labeling
- ... and many more!

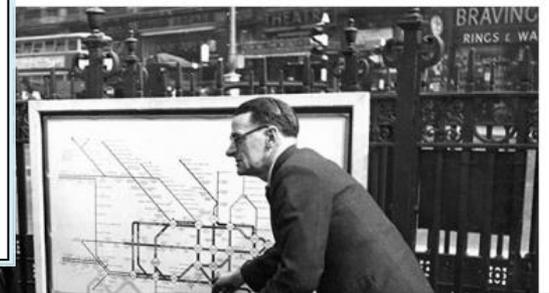
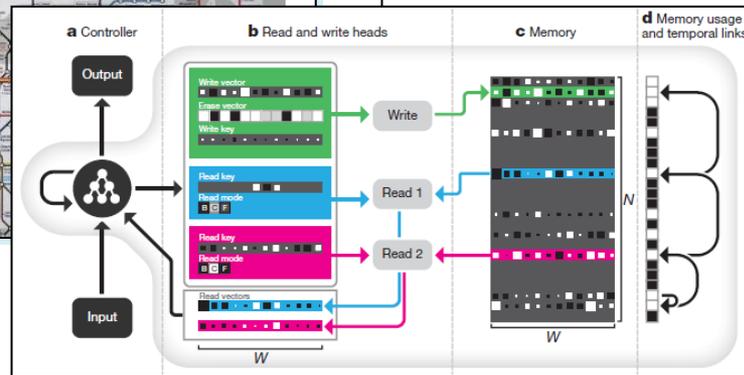
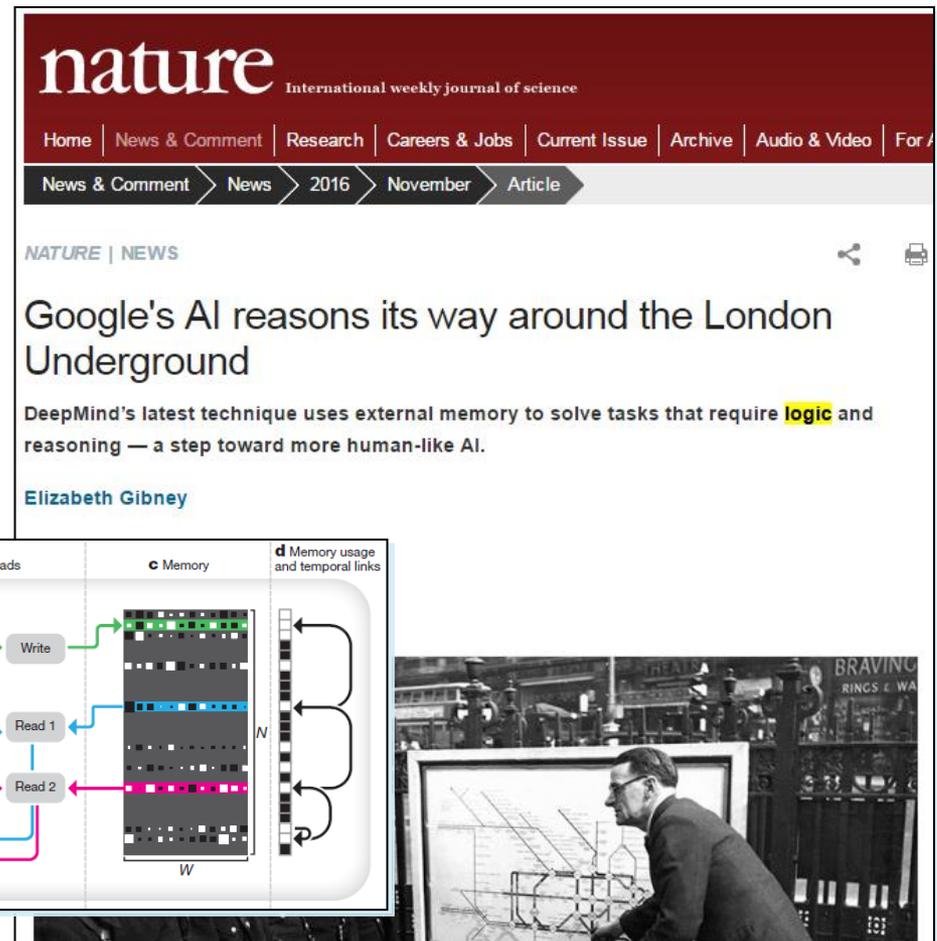
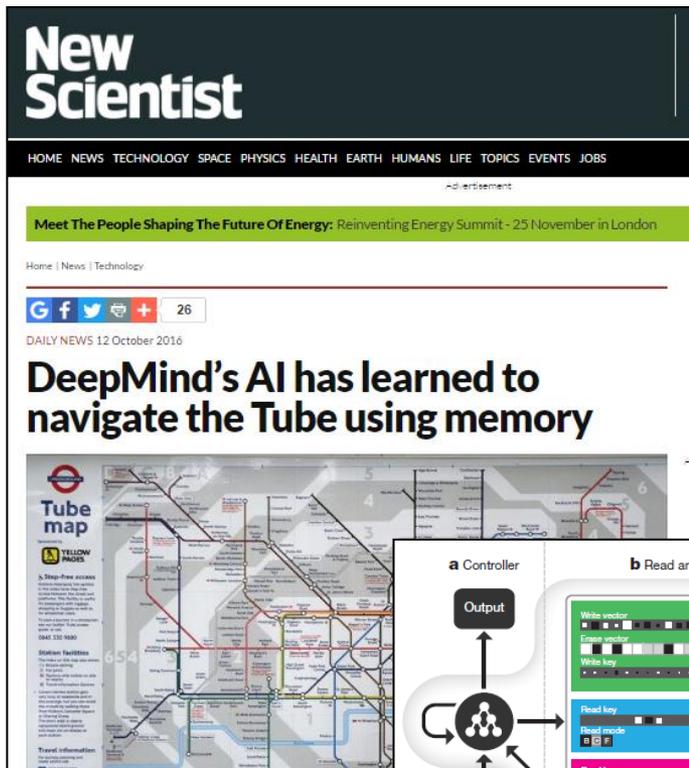
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Bayesian network synthesized from specs of power system (NASA Ames):
 Has many constraints (0/1 parameters) due to domain "physics"

- Query Mode - [C:\Docum
- adaptkind
- sensor**
- SensorCurrent_w0
 - readCurrentLo
 - readCurrentHi
 - SensorTouch_c0
 - readOpen
 - readClosed
 - SensorVoltage_w0
 - readVoltageLo
 - readVoltageHi
- command**
- Command_c0
 - cmdOpen
 - cmdClose
- health**
- Health_b0
 - Health_c0
 - Health_I0
 - SensorCurrentHealth_y
 - SensorTouchHealth_c0
 - SensorVoltageHealth_y
- current**
- Current_b0
 - Current_c0
 - Current_I0
 - Current_w0
- aux**
- OpenOrClosed_c0
 - OpenOrClosed_w0
 - Operational_b0
 - Operational_I0
 - ToBattery_b0
 - ToBattery_c0
 - ToBattery_I0
 - ToBattery_w0
 - ToLoad_b0

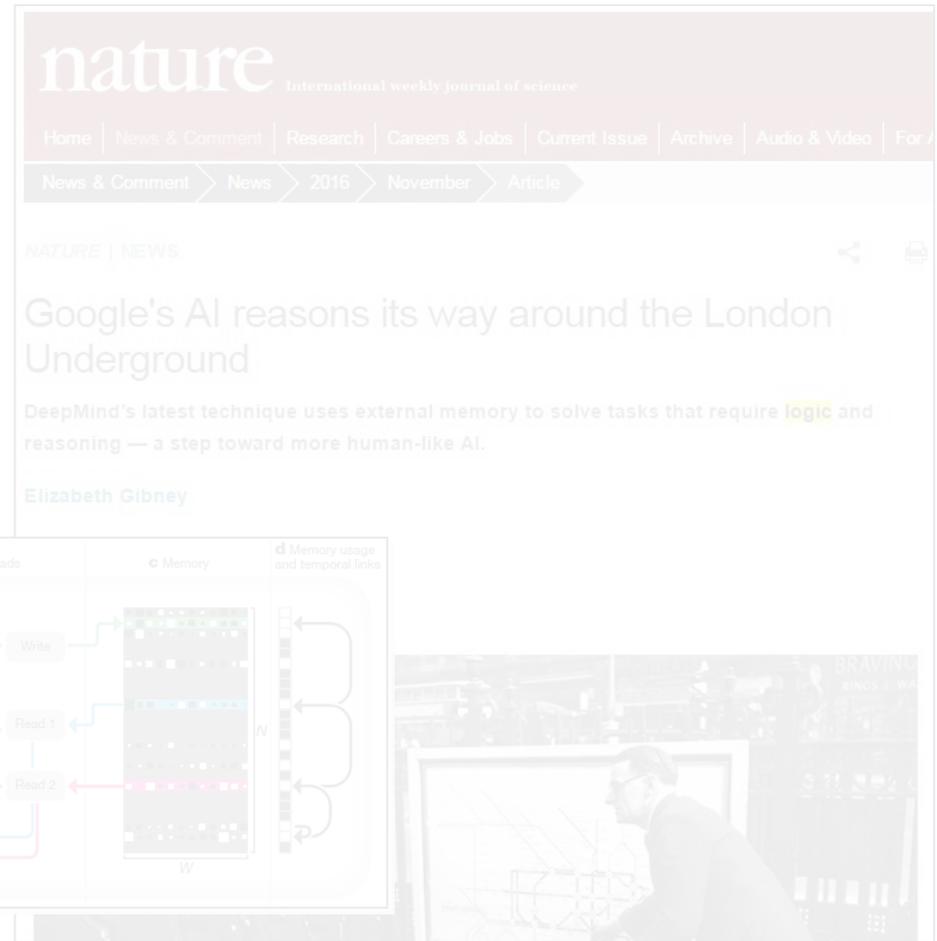
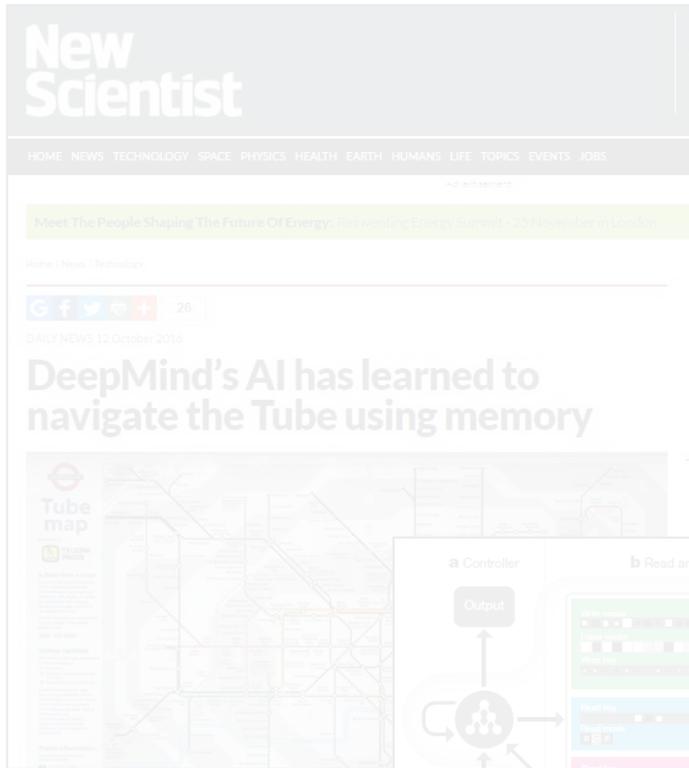


Example: Deep Learning



[Graves, A., Wayne, G., Reynolds, M., Harley, T., Danihelka, I., Grabska-Barwińska, A., et al.. (2016). Hybrid computing using a neural network with dynamic external memory. *Nature*, 538(7626), 471-476.]

Example: Deep Learning



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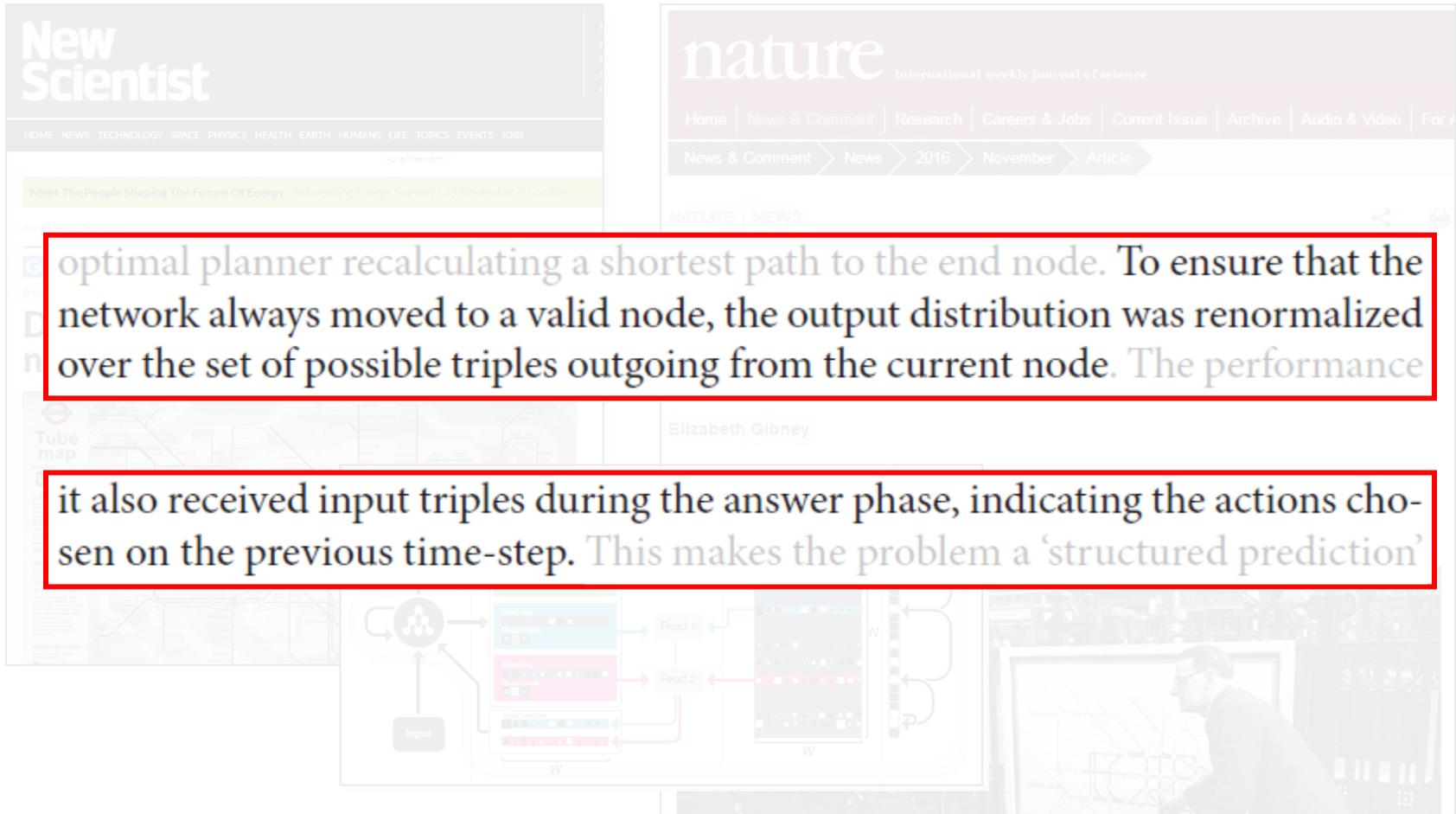
Example: Deep Learning

The collage features several elements: the top left shows the New Scientist magazine cover; the top right shows the Nature magazine cover; the middle left is a Tube map; the middle right shows a person at a computer; and the bottom center is a diagram of a neural network with dynamic external memory. The diagram is divided into four parts: a Controller with Input and Output, Read and write heads (Write, Read 1, Read 2), Memory (W), and Memory usage and temporal links (N).

optimal planner recalculating a shortest path to the end node. To ensure that the network always moved to a valid node, the output distribution was renormalized over the set of possible triples outgoing from the current node. The performance

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Example: Deep Learning



The background features several elements: the New Scientist website header, the Nature website header, a Tube map, and a diagram of a neural network with dynamic external memory. The diagram shows an input feeding into a neural network with layers labeled 'Read 1' and 'Read 2', which are connected to a memory structure labeled 'W'. The text is overlaid on these images.

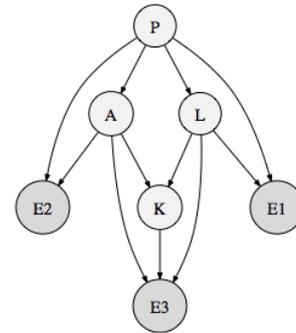
optimal planner recalculating a shortest path to the end node. To ensure that the network always moved to a valid node, the output distribution was renormalized over the set of possible triples outgoing from the current node. The performance

it also received input triples during the answer phase, indicating the actions chosen on the previous time-step. This makes the problem a 'structured prediction'

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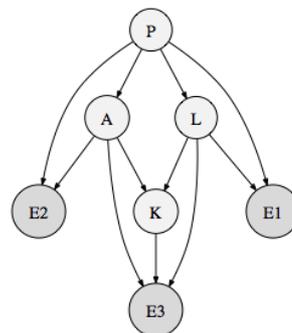
What are people doing now?

- Ignore constraints
- Handcraft into models
- Use specialized distributions
- Find non-structured encoding
- Try to learn constraints
- Hack your way around



What are people doing now?

- Ignore constraints
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Accuracy ?
Specialized skill ?
Intractable inference ?
Intractable learning ?
Waste parameters ?
Risk predicting out of space ?

you are on your own ☹️

Structured Probability Spaces

- Everywhere in ML!
 - Configuration problems, inventory, video, text, deep learning
 - Planning and diagnosis (physics)
 - Causal models: cooking scenarios (interpreting videos)
 - Combinatorial objects: parse trees, rankings, directed acyclic graphs, trees, simple paths, game traces, etc.

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No statistical ML boxes out there that take constraints as input! ☹️

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No statistical ML boxes out there that take constraints as input! ☹

Goal: Constraints as important as data! General purpose!

Specification Language: Logic

Structured Probability Space

unstructured

L	K	P	A
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1



structured

L	K	P	A
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
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- Must take at least one of Probability or Logic.
- Probability is a prerequisite for AI.
- The prerequisites for KR is either AI or Logic.

**7 out of 16 instantiations
are impossible**

Boolean Constraints

unstructured

L	K	P	A
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1



structured

L	K	P	A
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

$$\begin{aligned} P \vee L \\ A \Rightarrow P \\ K \Rightarrow (P \vee L) \end{aligned}$$

**7 out of 16 instantiations
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Combinatorial Objects: Rankings

rank	sushi
1	fatty tuna
2	sea urchin
3	salmon roe
4	shrimp
5	tuna
6	squid
7	tuna roll
8	see eel
9	egg
10	cucumber roll

rank	sushi
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2	sea urchin
3	salmon roe
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5	tuna
6	squid
7	tuna roll
8	see eel
9	egg
10	cucumber roll

10 items:
3,628,800
rankings

20 items:
2,432,902,008,176,640,000
rankings

Combinatorial Objects: Rankings

rank	sushi
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A_{ij} item i at position j
(n items require n^2
Boolean variables)

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A_{ij} item i at position j
(n items require n^2
Boolean variables)

An item may be assigned
to more than one position

A position may contain
more than one item

Encoding Rankings in Logic

A_{ij} : item i at position j

	pos 1	pos 2	pos 3	pos 4
item 1	A_{11}	A_{12}	A_{13}	A_{14}
item 2	A_{21}	A_{22}	A_{23}	A_{24}
item 3	A_{31}	A_{32}	A_{33}	A_{34}
item 4	A_{41}	A_{42}	A_{43}	A_{44}

Encoding Rankings in Logic

A_{ij} : item i at position j

	pos 1	pos 2	pos 3	pos 4
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item 3	A_{31}	A_{32}	A_{33}	A_{34}
item 4	A_{41}	A_{42}	A_{43}	A_{44}

constraint: each item i assigned to a unique position (n constraints)

$$\bigvee_j A_{ij} \wedge \left(\bigwedge_{k \neq j} \neg A_{ik} \right)$$

Encoding Rankings in Logic

A_{ij} : item i at position j

	pos 1	pos 2	pos 3	pos 4
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$$\bigvee_i A_{ij} \wedge \left(\bigwedge_{k \neq i} \neg A_{kj} \right)$$

Encoding Rankings in Logic

A_{ij} : item i at position j

	pos 1	pos 2	pos 3	pos 4
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item 4	A_{41}	A_{42}	A_{43}	A_{44}

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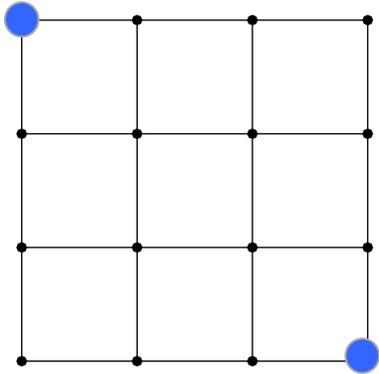
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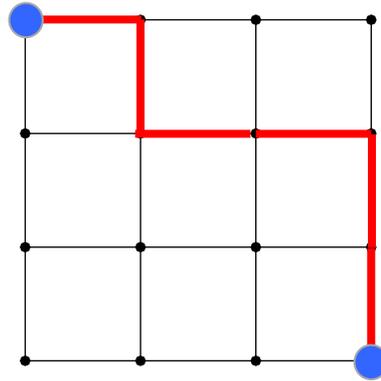
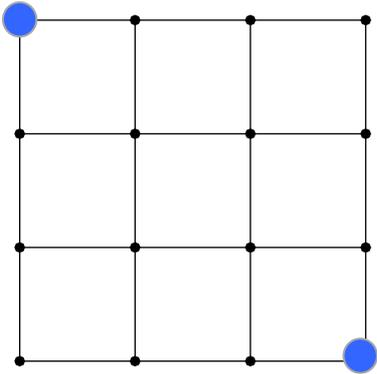
$$\bigvee_i A_{ij} \wedge \left(\bigwedge_{k \neq i} \neg A_{kj} \right)$$

total constraints $2n$
unstructured space 2^{n^2}
structured space $n!$

Structured Space for Paths



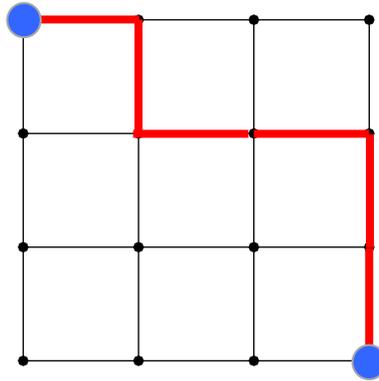
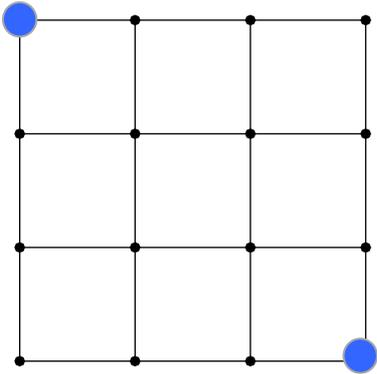
Structured Space for Paths



**Good variable assignment
(represents route)**

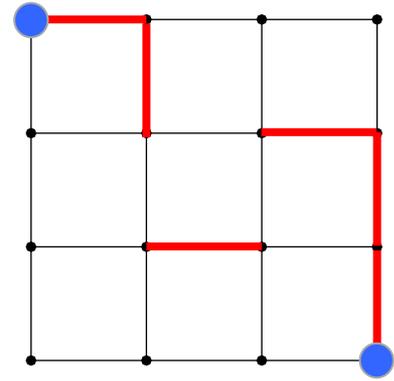
184

Structured Space for Paths



**Good variable assignment
(represents route)**

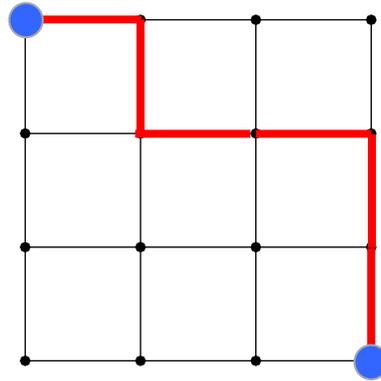
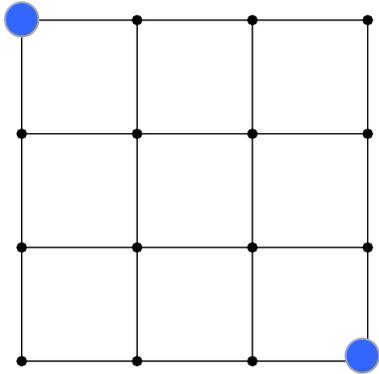
184



**Bad variable assignment
(does not represent route)**

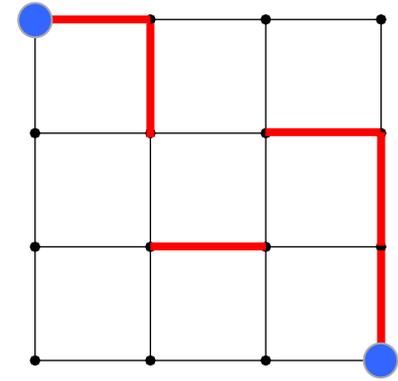
16,777,032

Structured Space for Paths



**Good variable assignment
(represents route)**

184

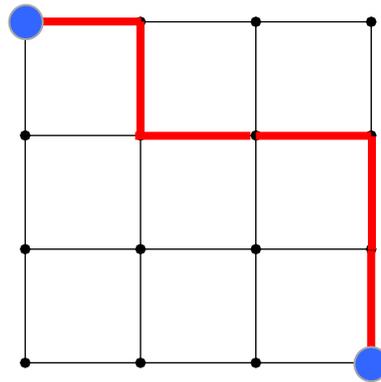
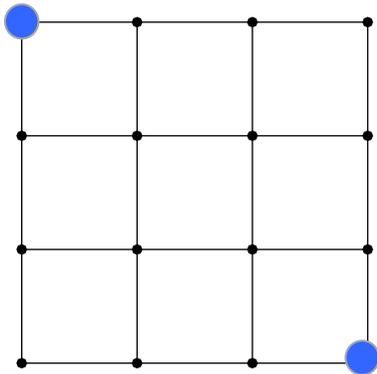


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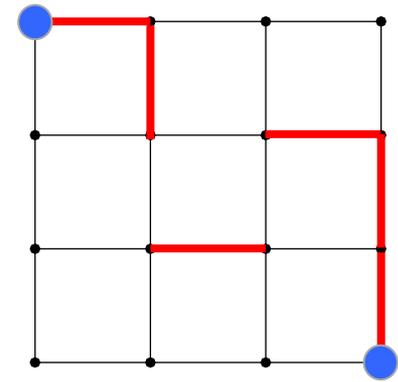
Space easily encoded in logical constraints 😊

Structured Space for Paths



Good variable assignment
(represents route)

184



Bad variable assignment
(does not represent route)

16,777,032

Space easily encoded in logical constraints 😊

Unstructured probability space: $184 + 16,777,032 = 2^{24}$

Undirected Graphs (Unstructured)

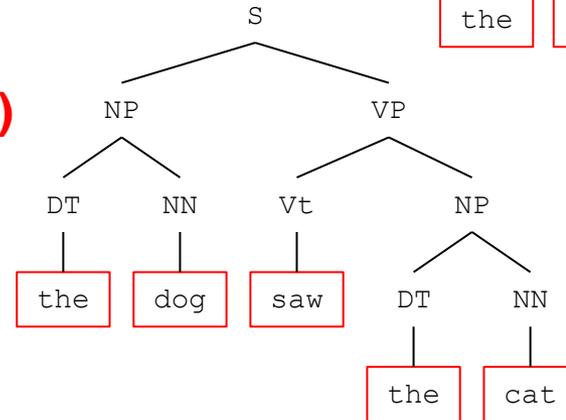
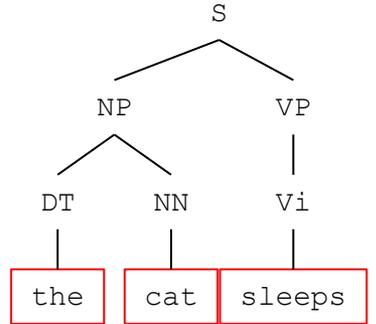
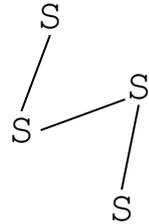
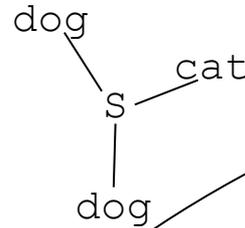
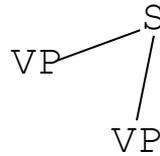
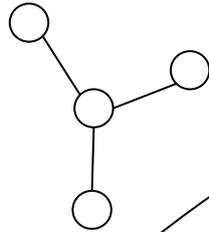
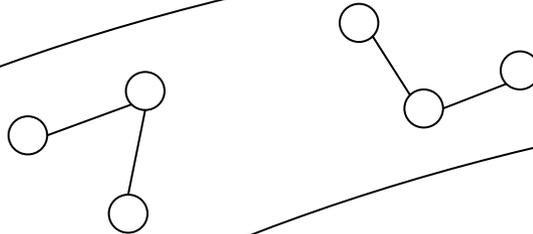
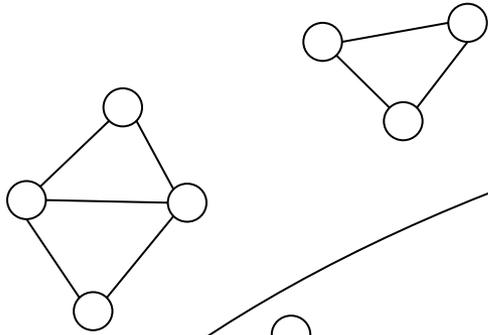
Trees

Labeled Trees

Parse Trees

Acyclicity Constraints

**Label Constraints
(CFG Production Rules)**

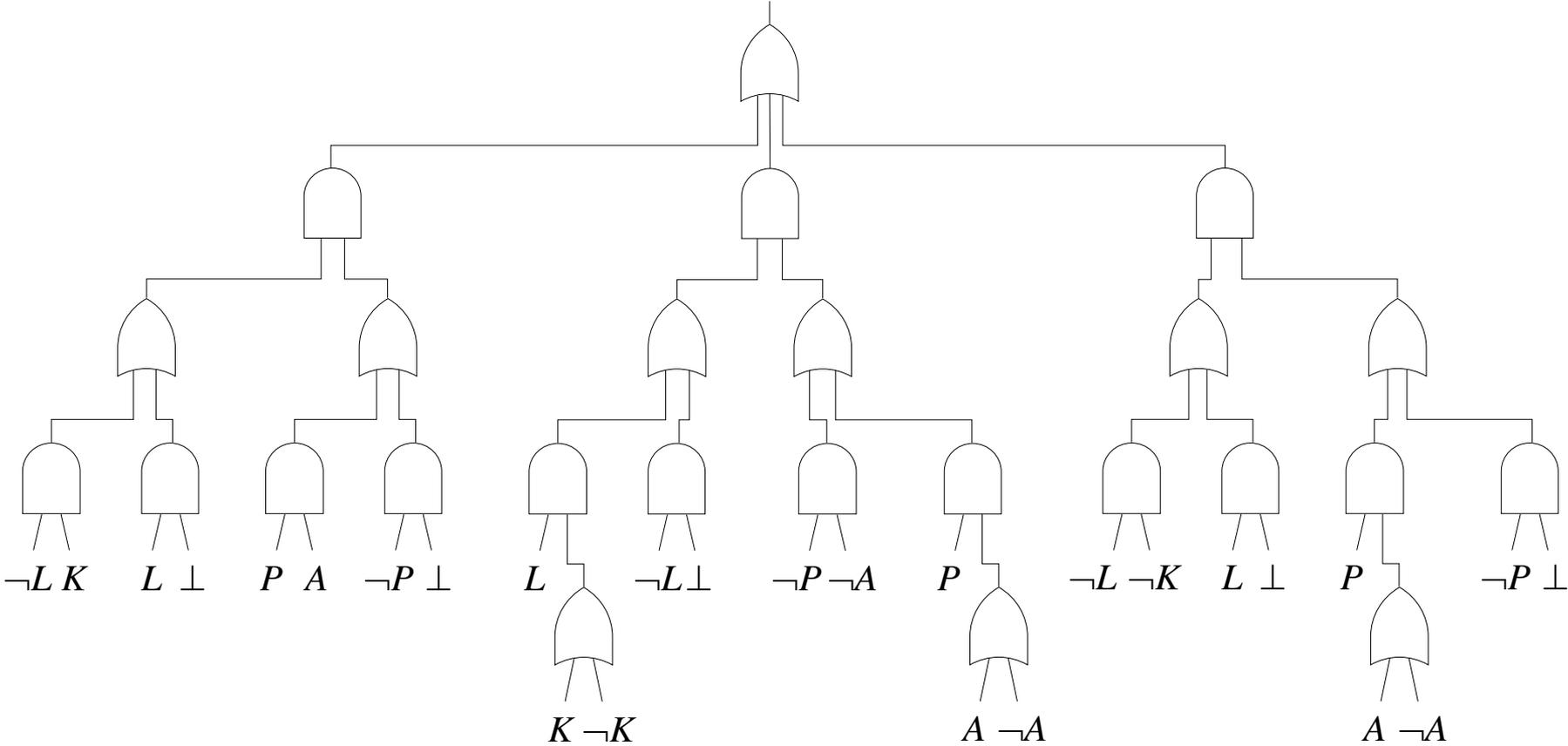


“Deep Architecture”

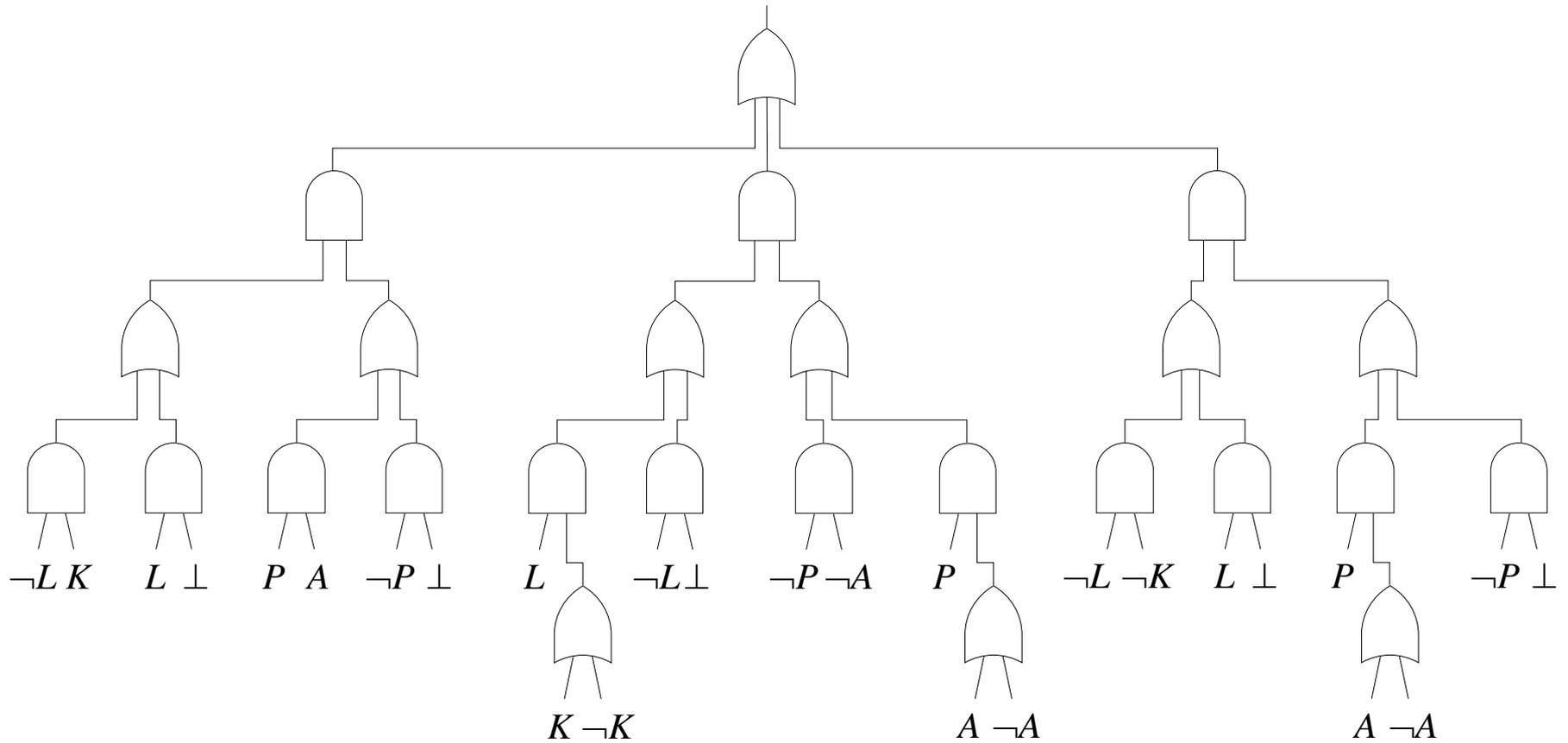
Logic + Probability

Logical Circuits

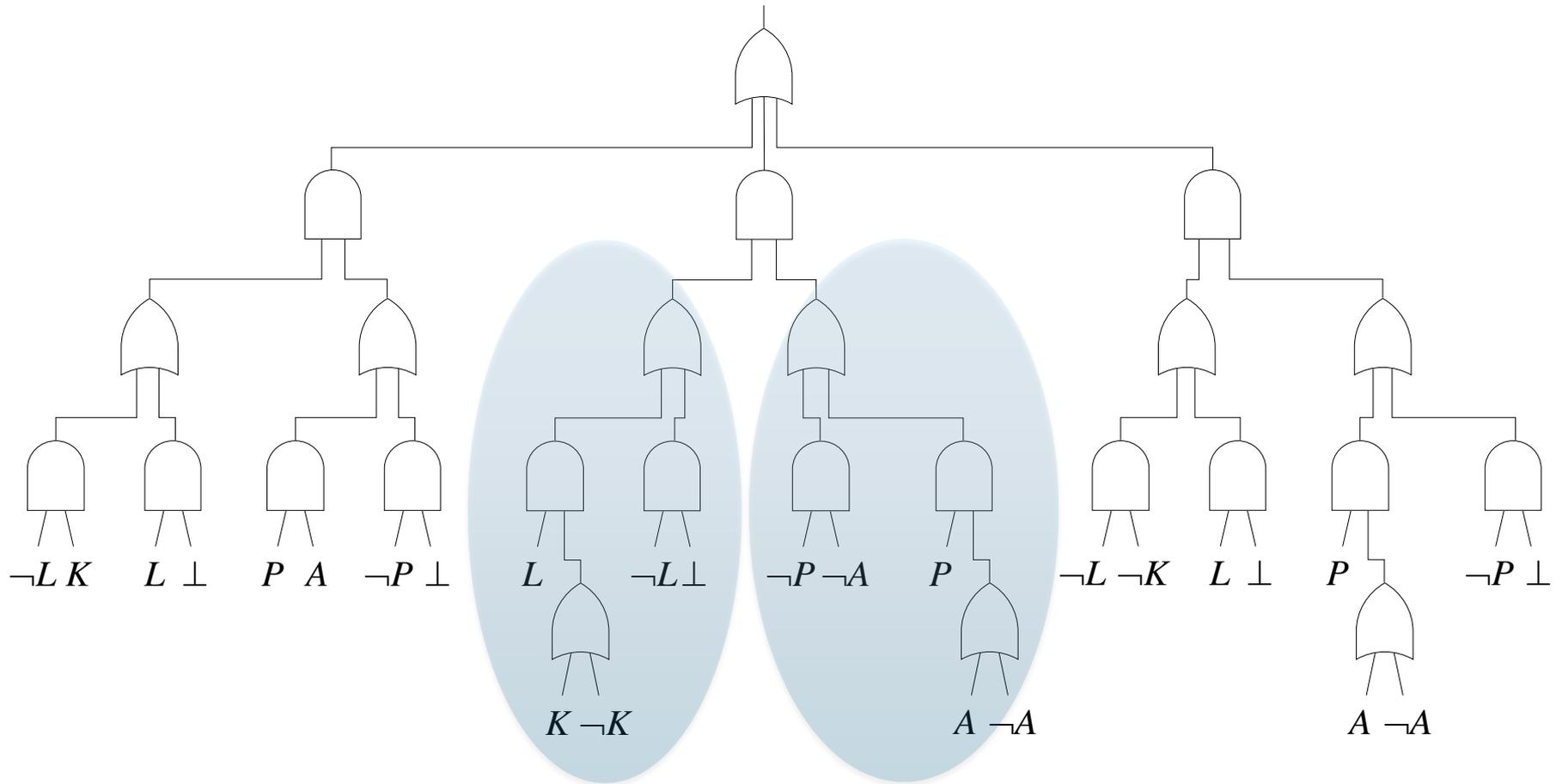
$P \vee L$
 $A \Rightarrow P$
 $K \Rightarrow (P \vee L)$



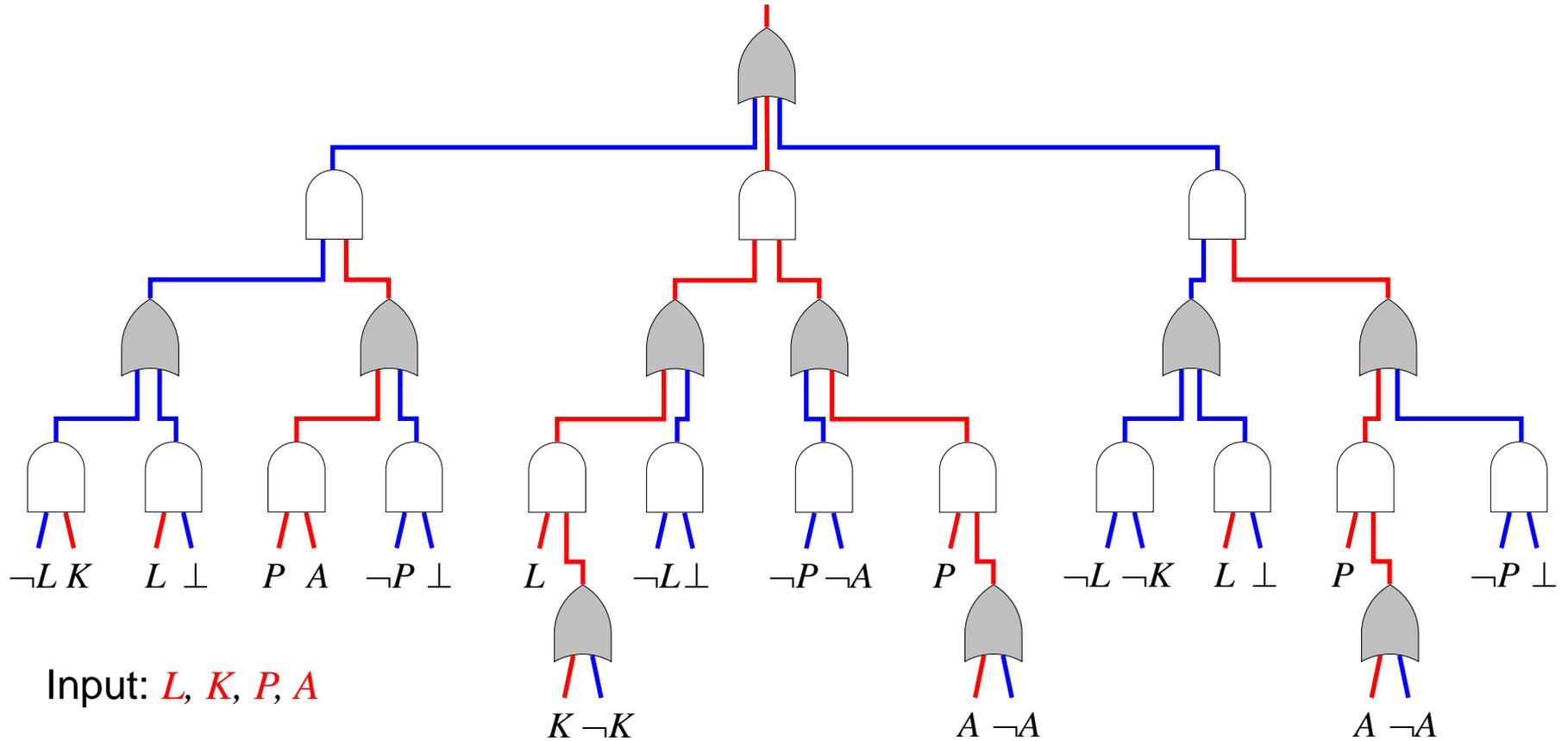
Property: Decomposability



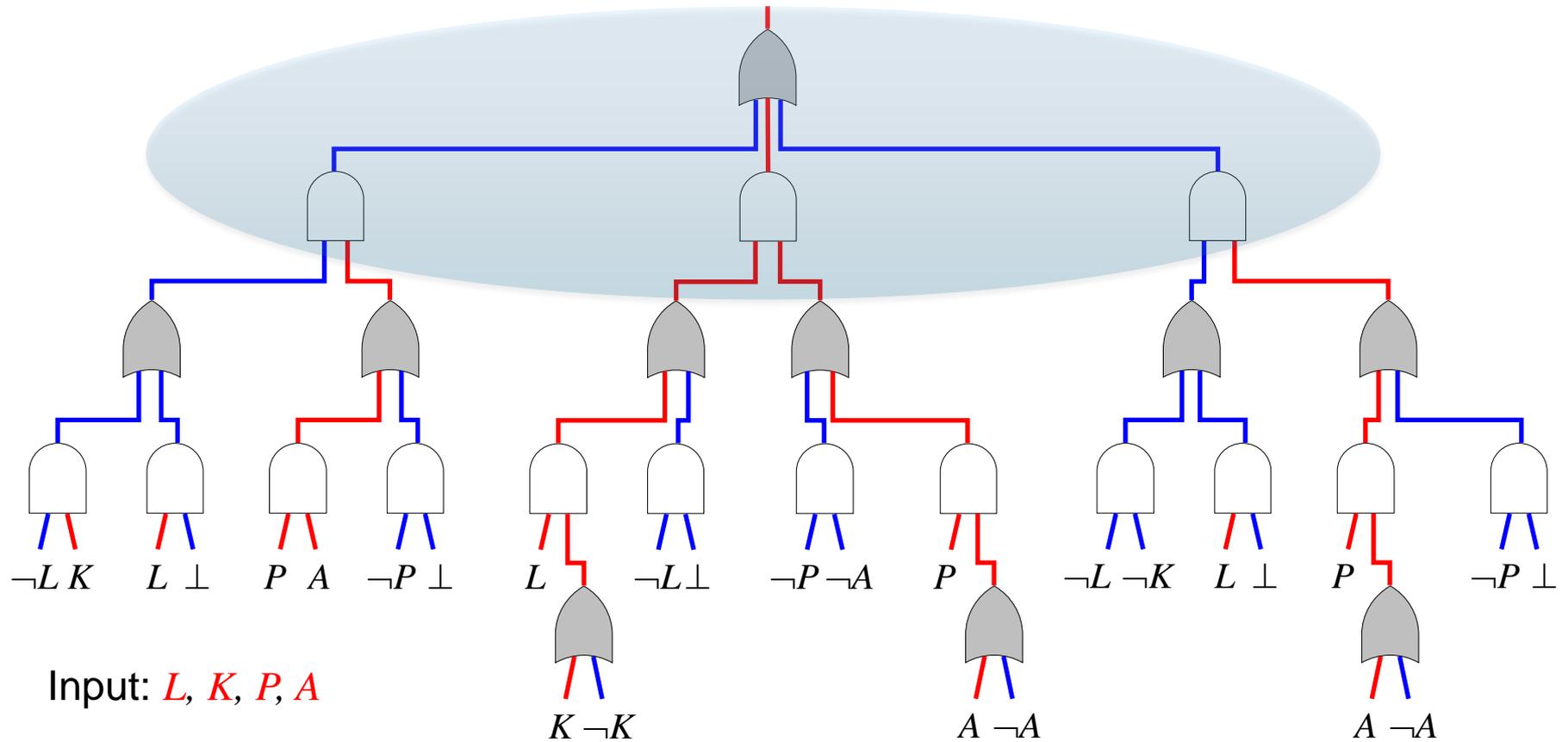
Property: Decomposability



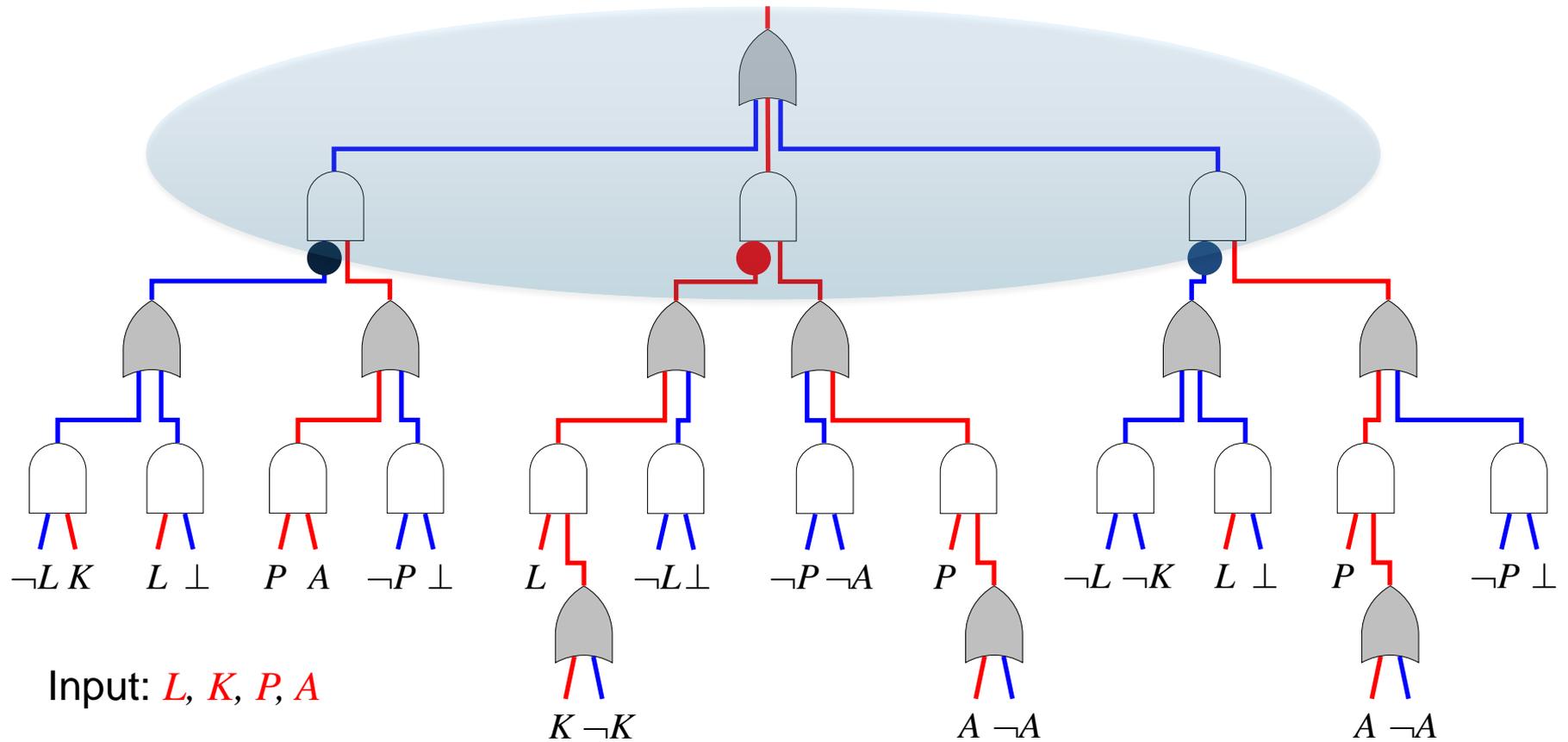
Property: Determinism



Sentential Decision Diagram (SDD)



Sentential Decision Diagram (SDD)



Tractable for Logical Inference

- Is structured space empty? (SAT)
- Count size of structured space (#SAT)
- Check equivalence of spaces

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Algorithms linear in circuit size 😊

(pass up, pass down, similar to backprop)

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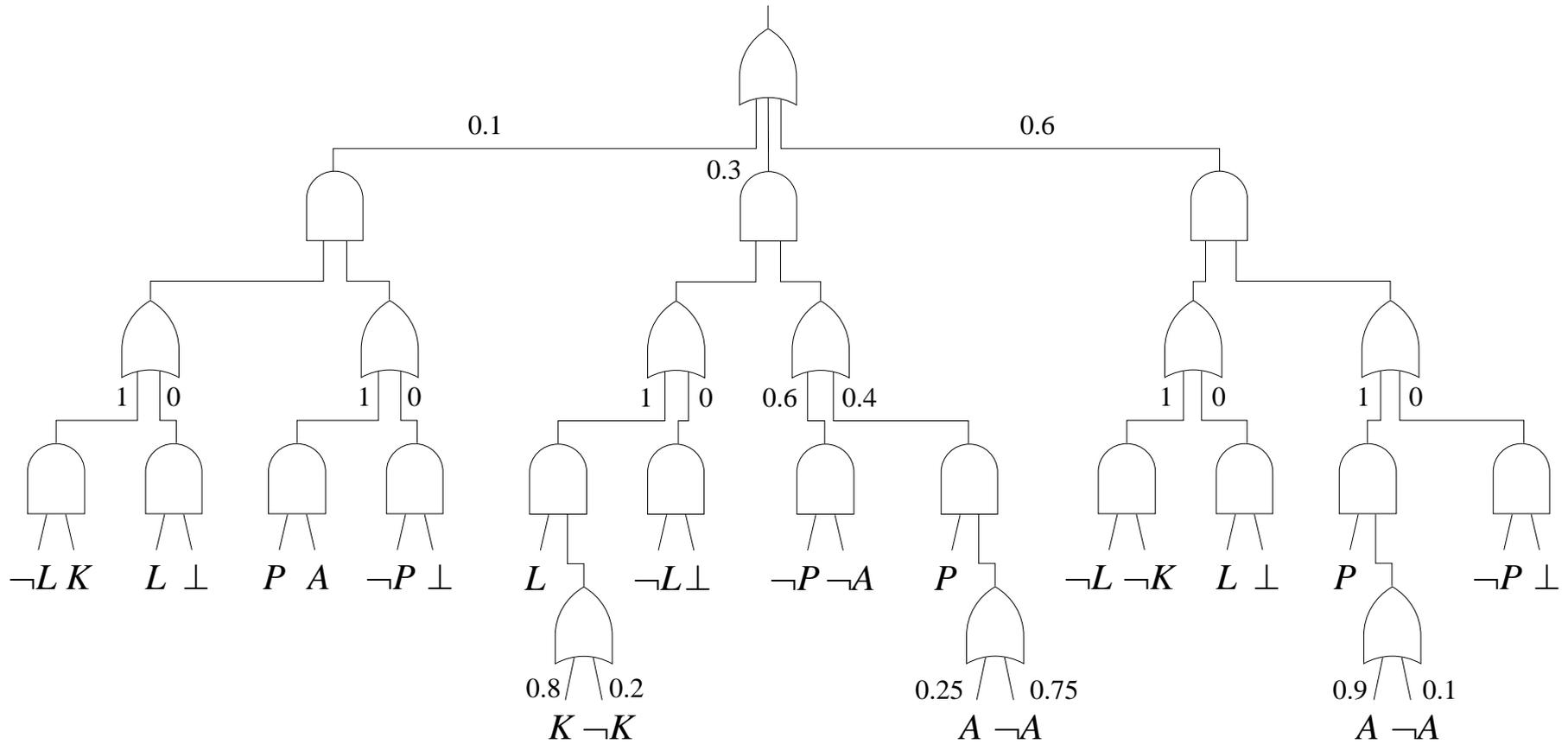
SCIENCE + TECHNOLOGY

Artificial intelligence framework developed by UCLA professor now powers Toyota websites

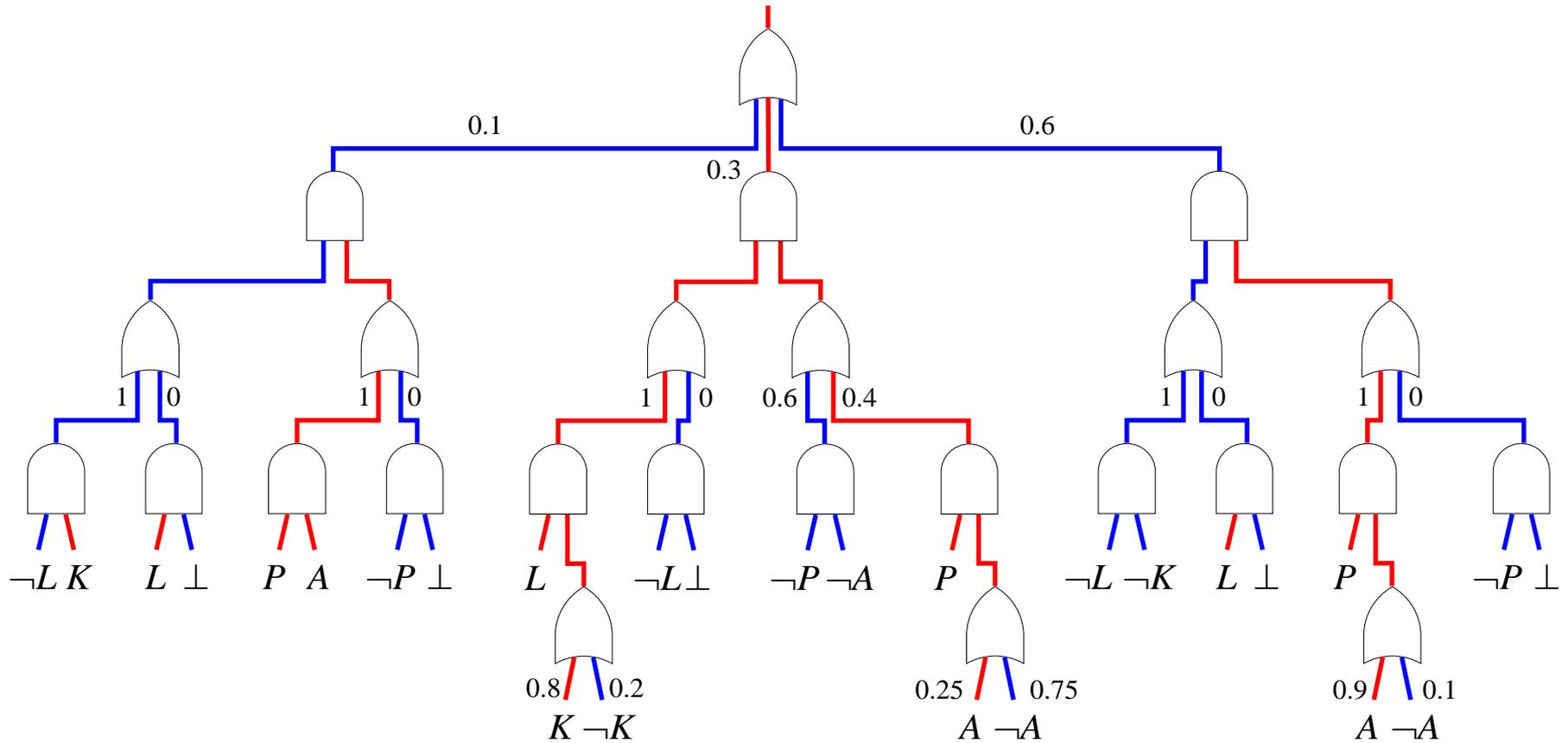
Adnan Darwiche's invention helps consumers customize their vehicles online

Matthew Chin | May 12, 2016

PSDD: Probabilistic SDD

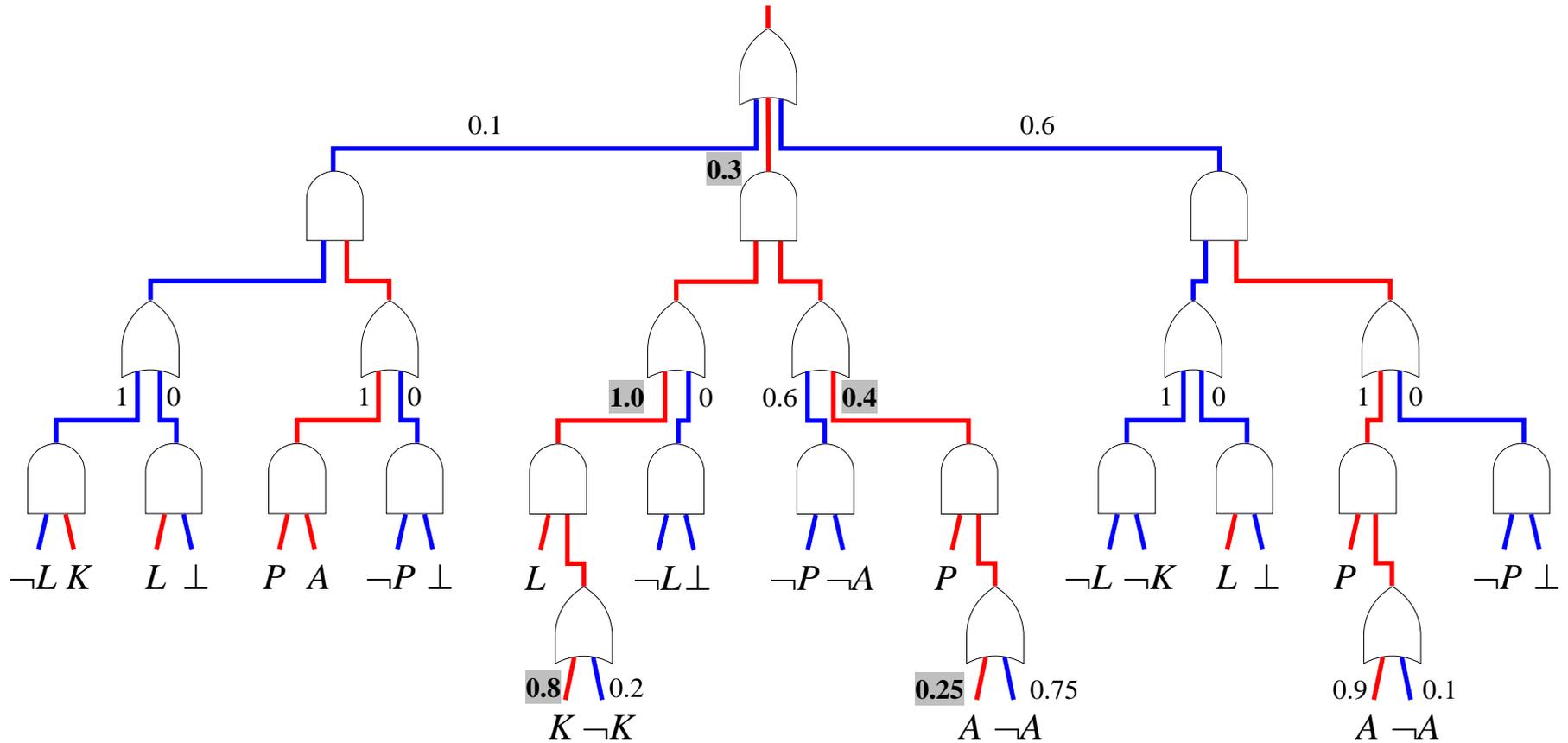


PSDD: Probabilistic SDD



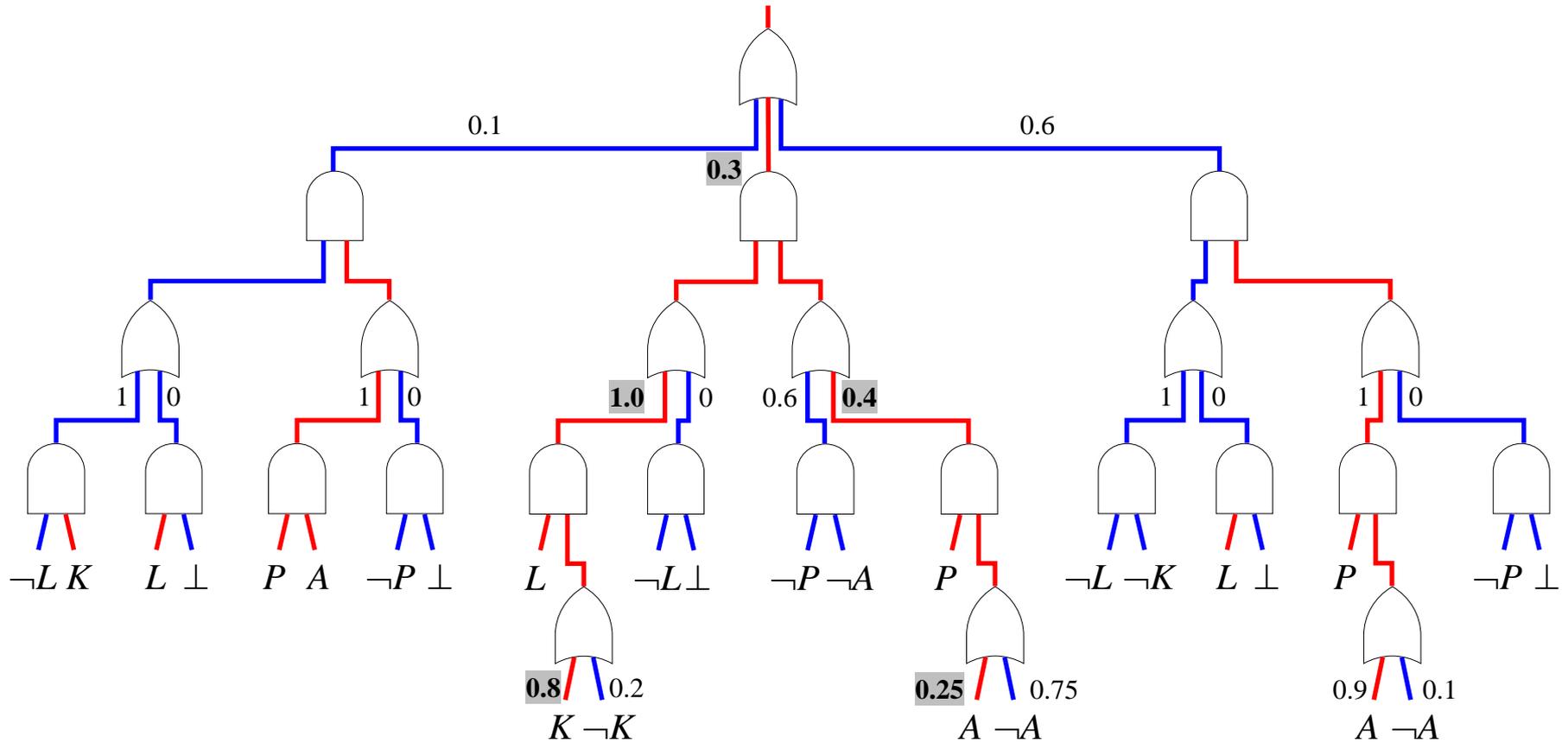
Input: L, K, P, A

PSDD: Probabilistic SDD



Input: L, K, P, A

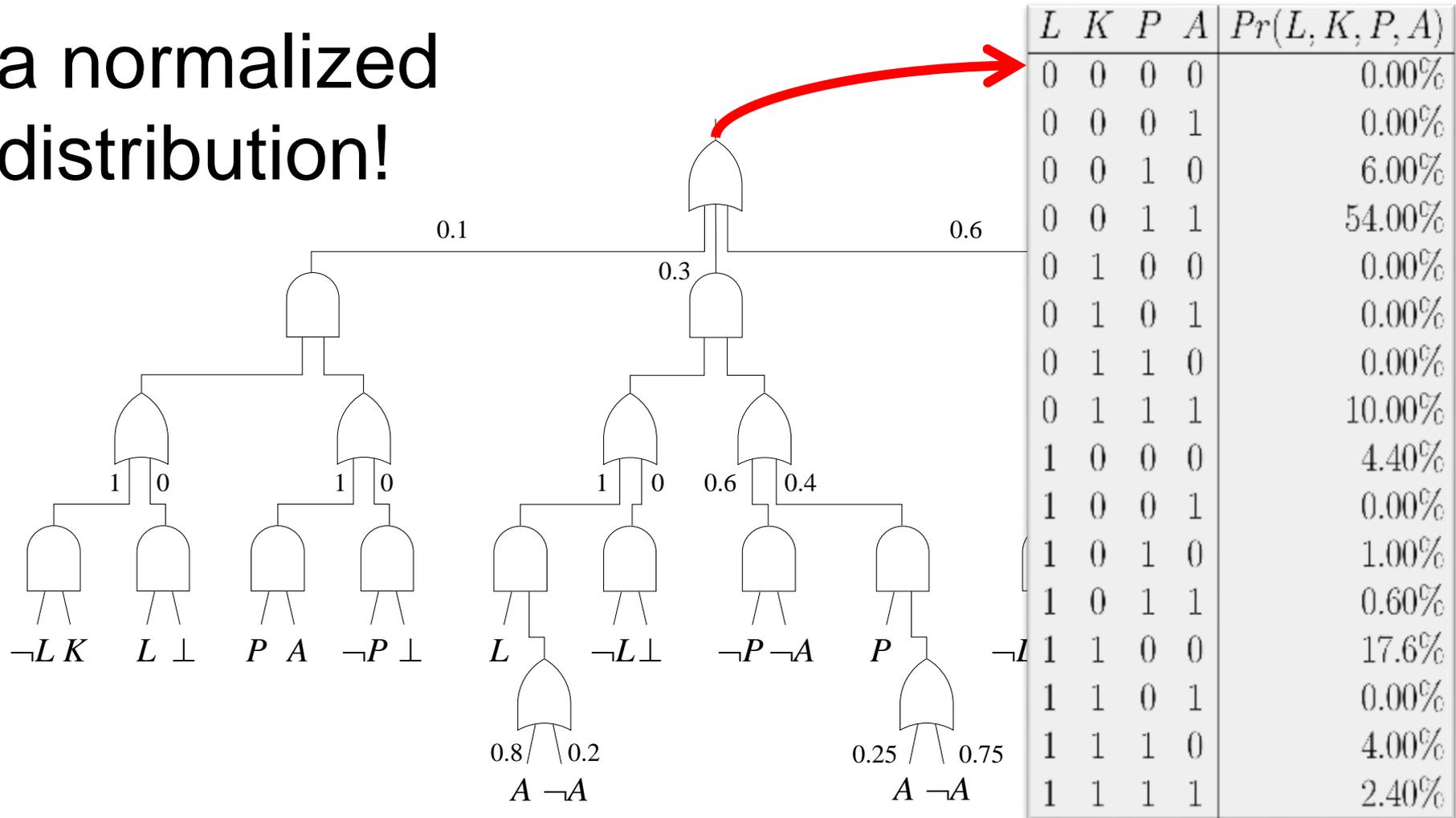
PSDD: Probabilistic SDD



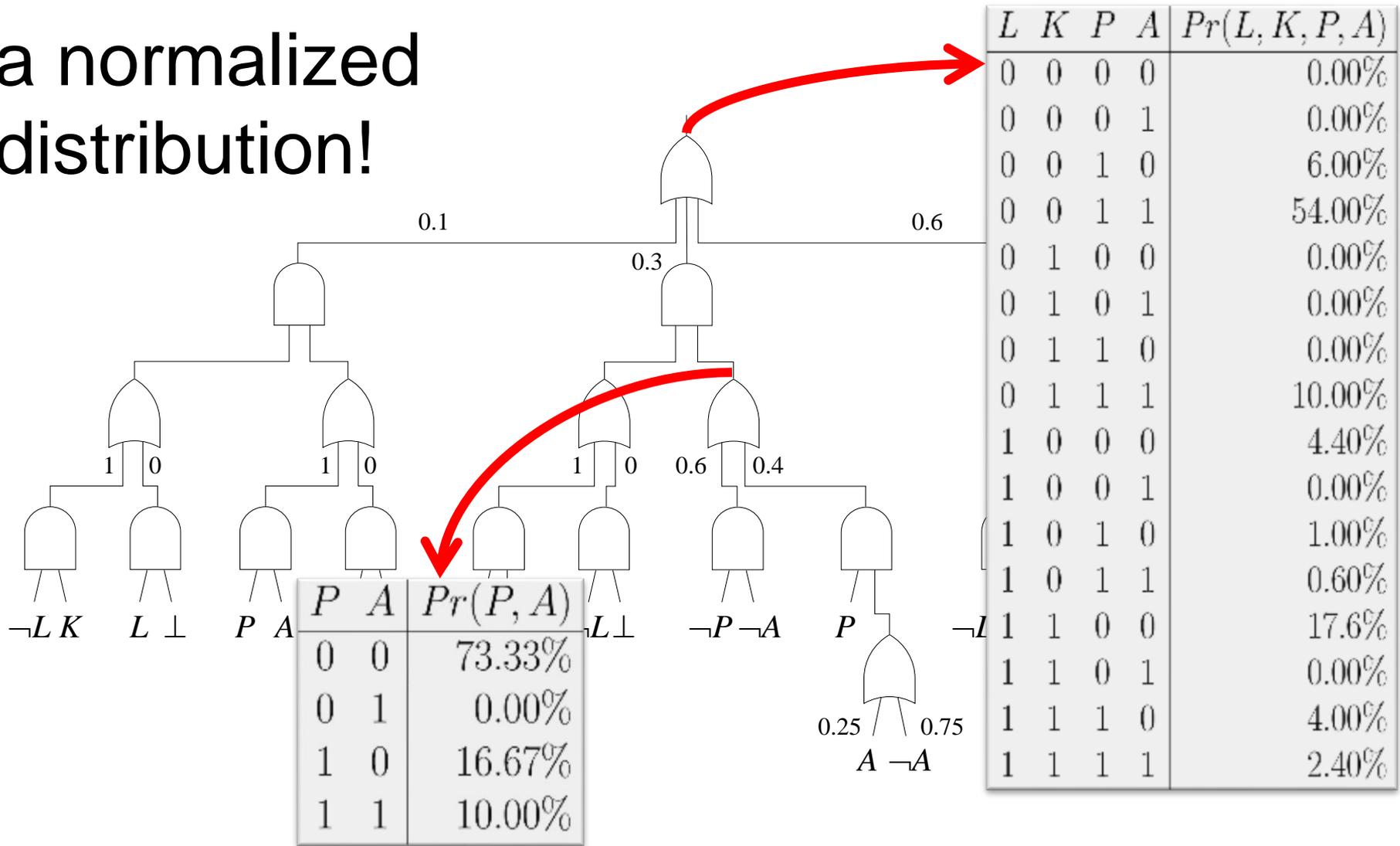
Input: L, K, P, A

$$\Pr(L, K, P, A) = 0.3 \times 1.0 \times 0.8 \times 0.4 \times 0.25 = 0.024$$

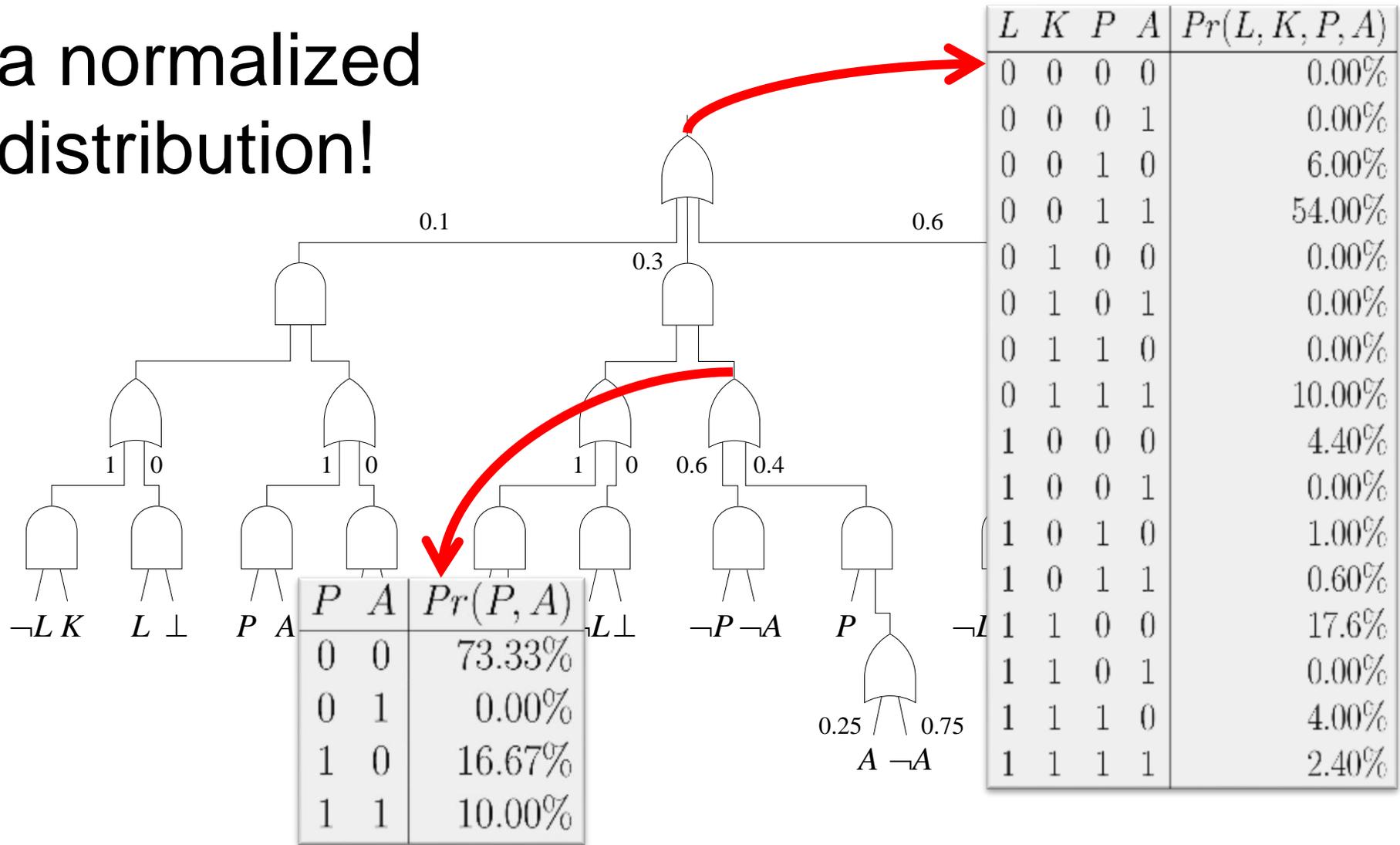
PSDD nodes induce a normalized distribution!



PSDD nodes induce a normalized distribution!



PSDD nodes induce a normalized distribution!



Can read probabilistic independences off the circuit structure

Tractable for Probabilistic Inference

- **MAP inference**: Find most-likely assignment (otherwise NP-complete)
- Computing **conditional probabilities** $\Pr(x|y)$ (otherwise PP-complete)
- **Sample** from $\Pr(x|y)$

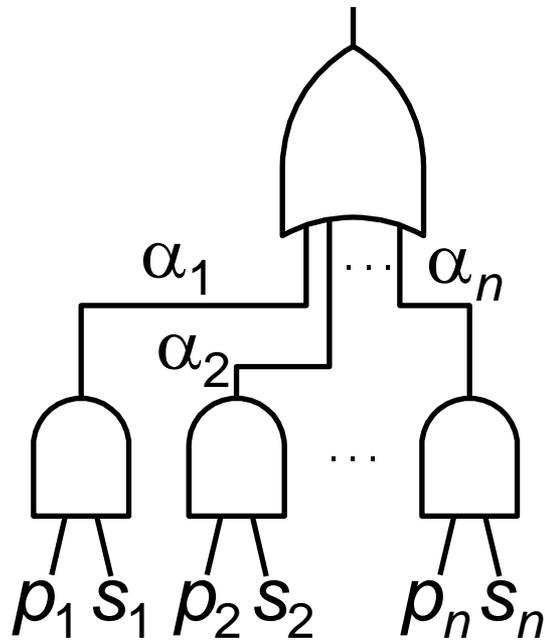
Tractable for Probabilistic Inference

- **MAP inference**: Find most-likely assignment (otherwise NP-complete)
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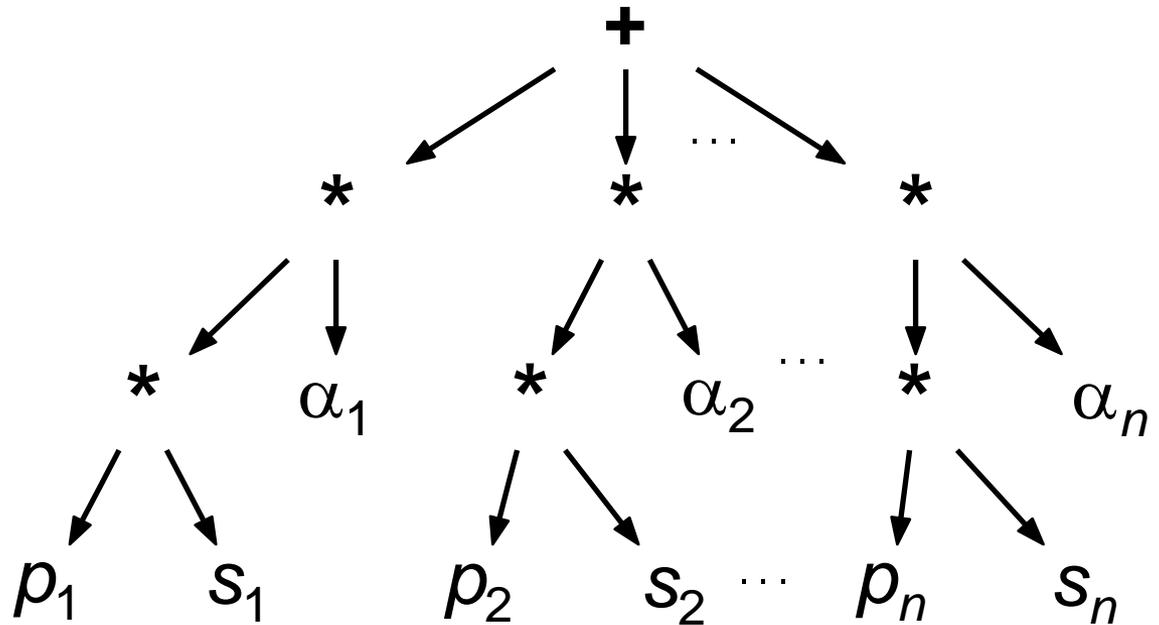
Algorithms linear in circuit size 😊
(pass up, pass down, similar to backprop)

PSDDs are Arithmetic Circuits

[Darwiche, JACM 2003]



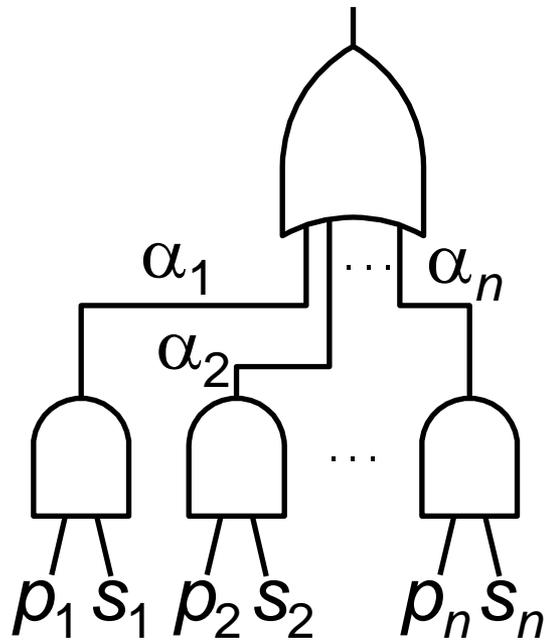
PSDD



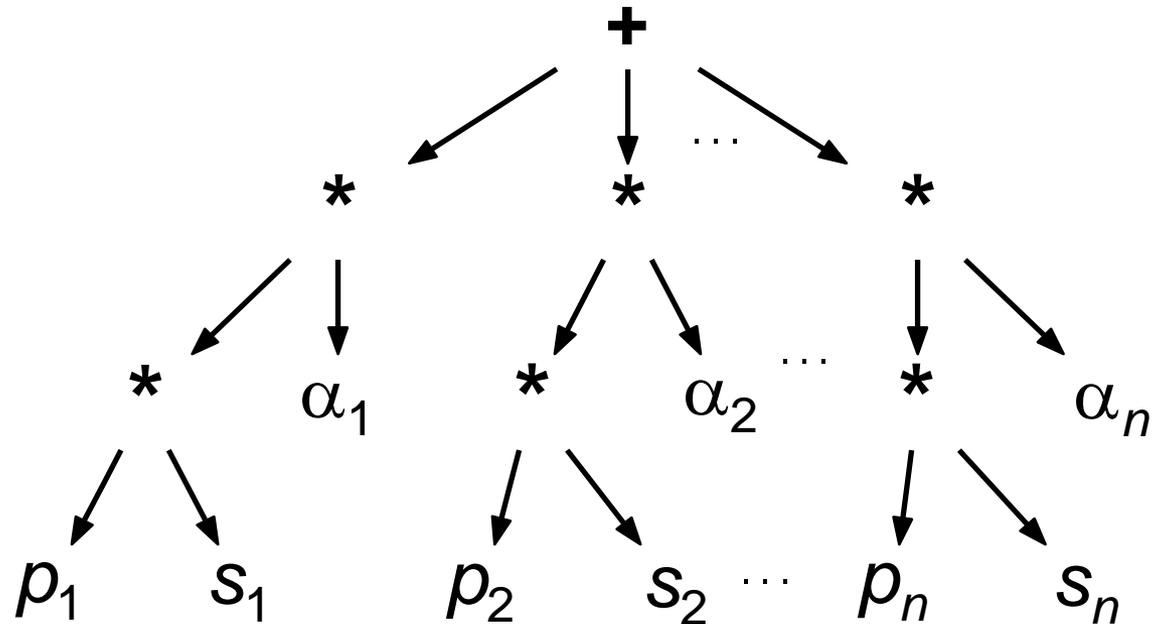
AC

PSDDs are Arithmetic Circuits

[Darwiche, JACM 2003]



PSDD



AC

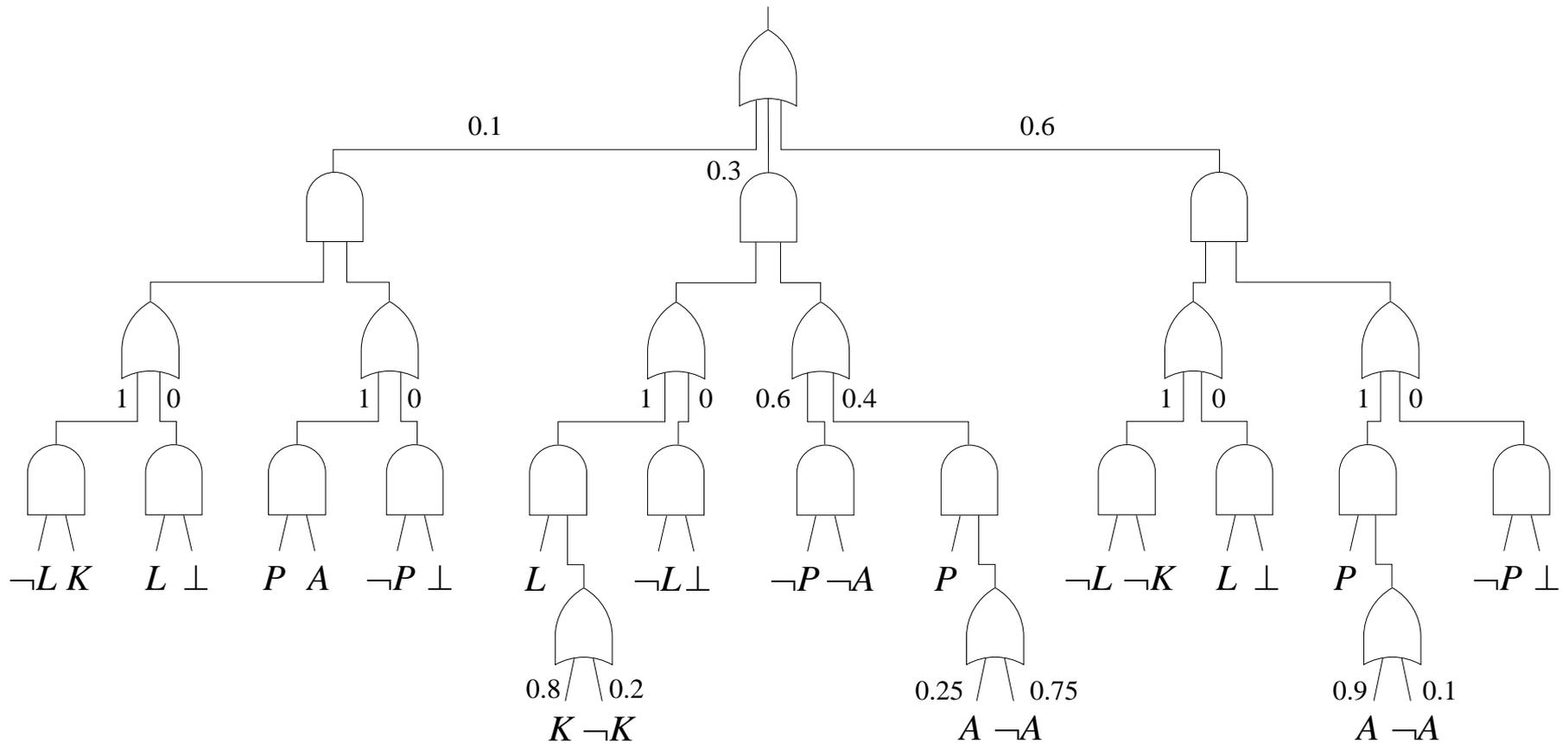
Known in the ML literature as SPNs
UAI 2011, NIPS 2012 best paper awards

[ICML 2014]
(SPNs equivalent to ACs)

Learning PSDDs

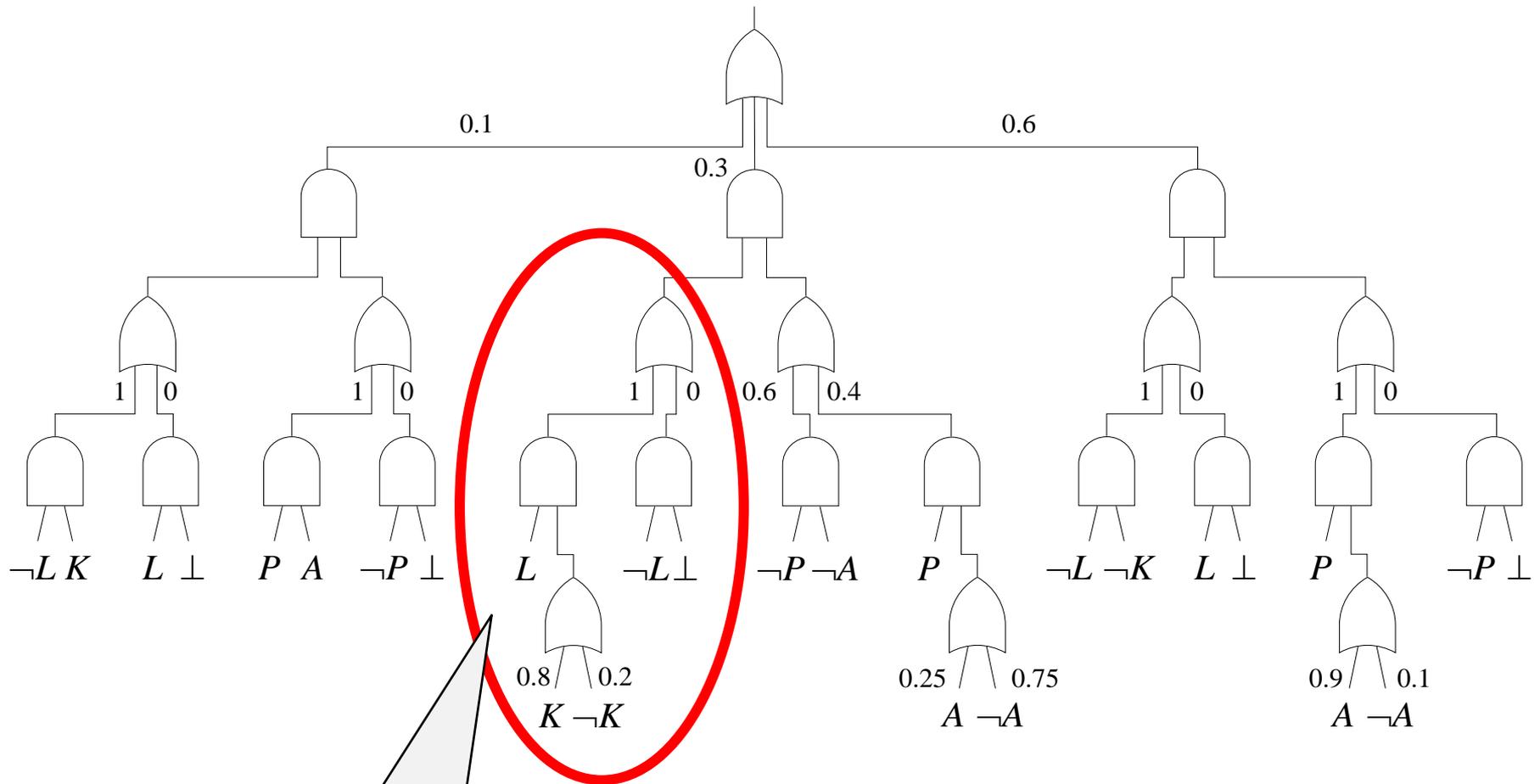
Logic + Probability + ML

Parameters are Interpretable



Explainable AI DARPA Program

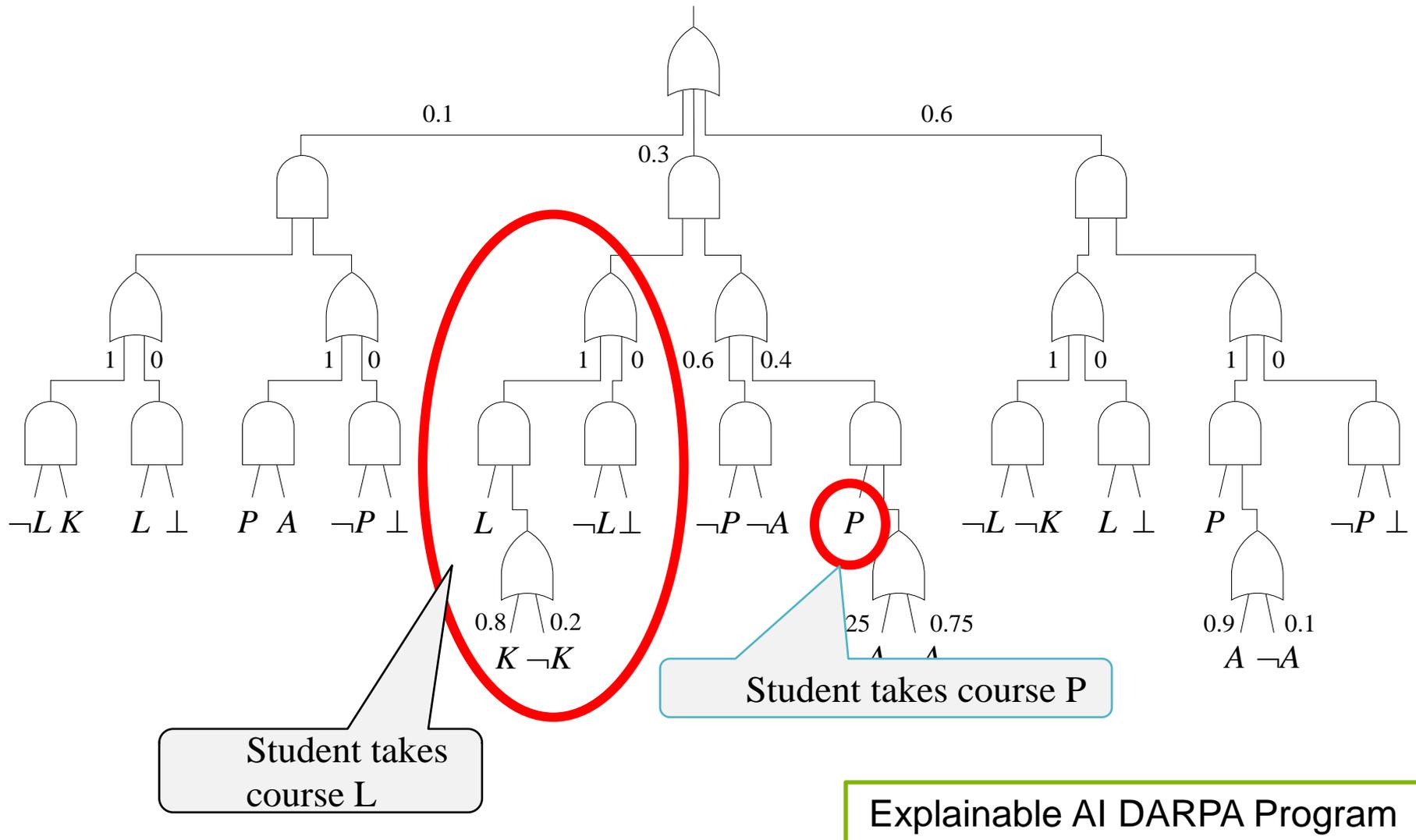
Parameters are Interpretable



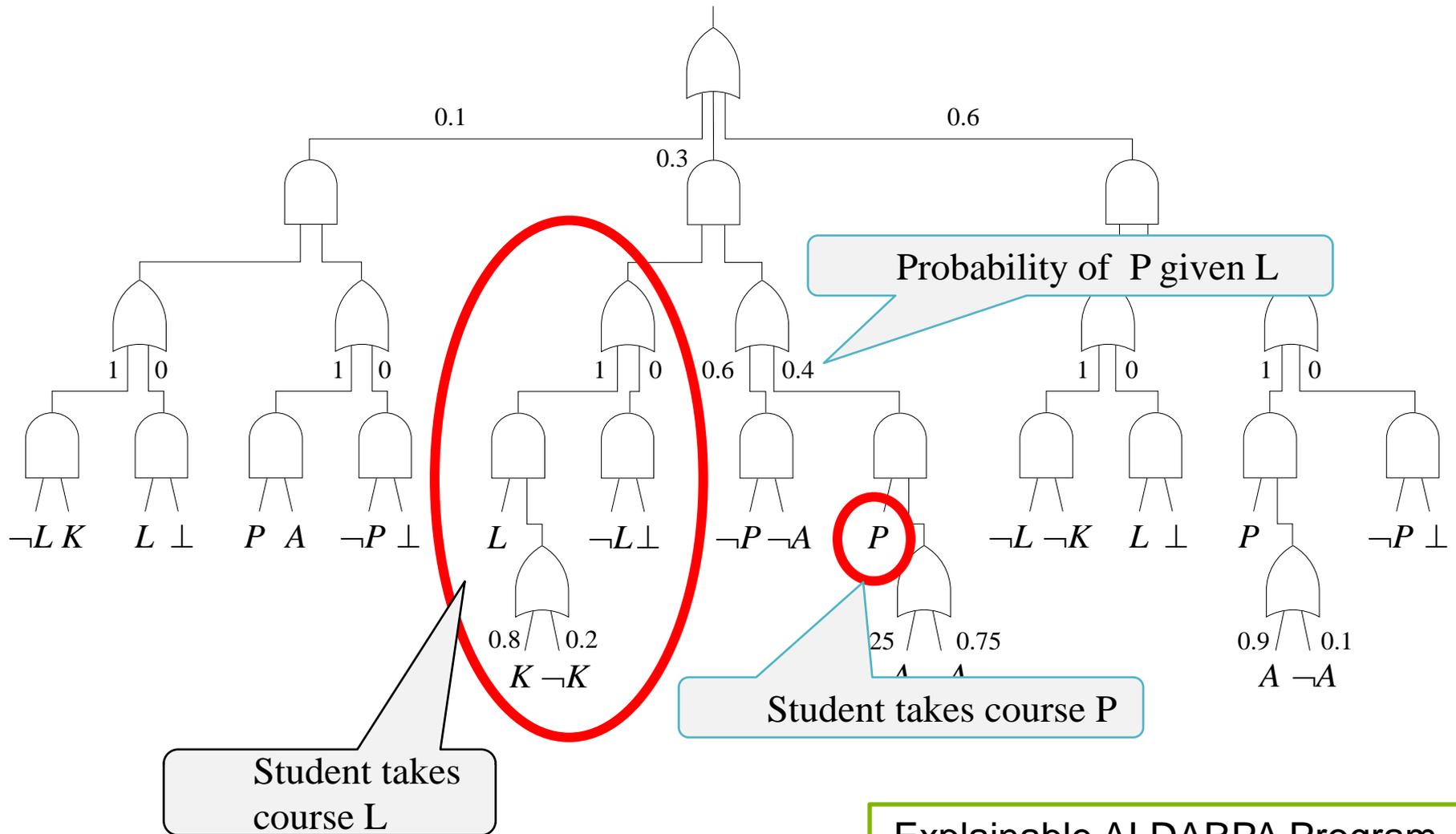
Student takes course L

Explainable AI DARPA Program

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Parameters are Interpretable



Learning Algorithms

- Parameter learning:
Closed form max likelihood from complete data
One pass over data to estimate $\Pr(x|y)$

Note a lot to say: very easy!

Learning Algorithms

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 - Compile constraints to SDD (naive)
 - Use SAT solver technology

Learning Algorithms

- Parameter learning:

Closed form max likelihood from complete data

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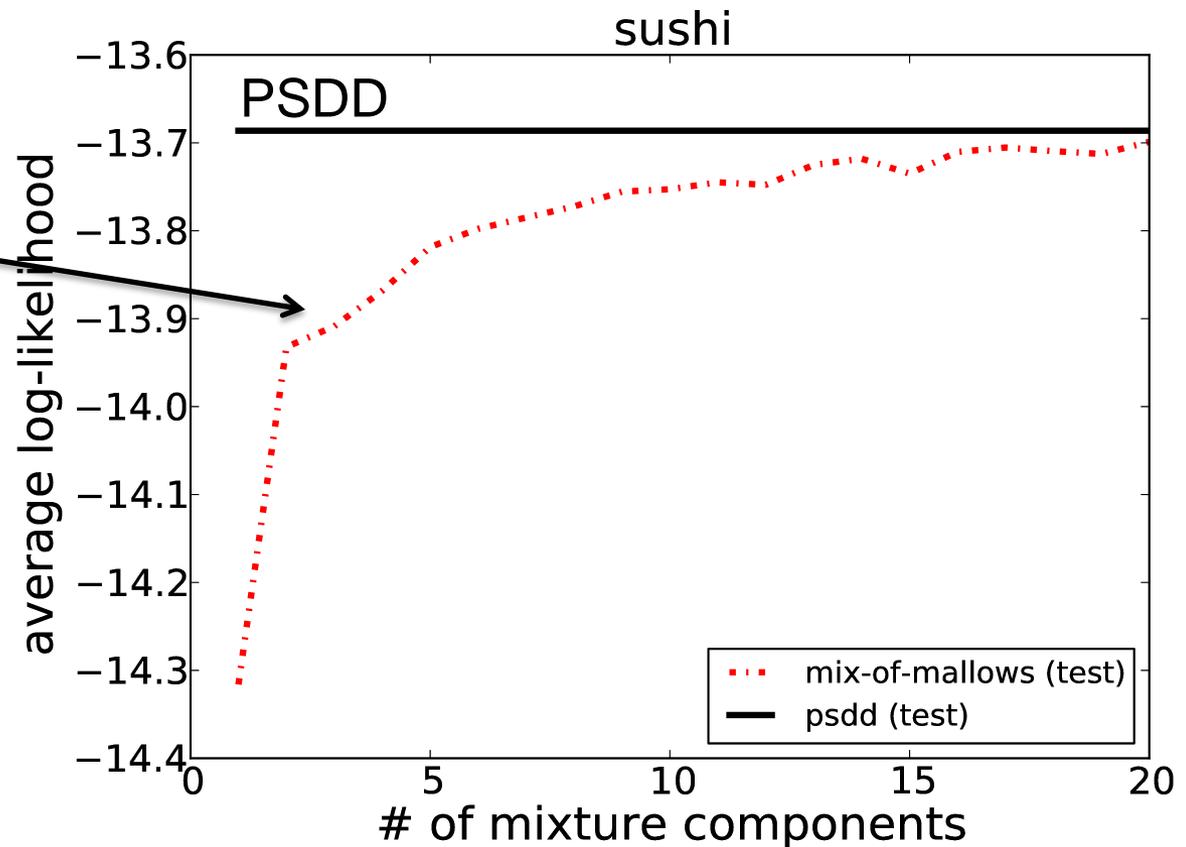
 - Use SAT solver technology

- Search for structure to fit data (ongoing work)

Learning Preference Distributions

Special-purpose
distribution:
Mixture-of-Mallows

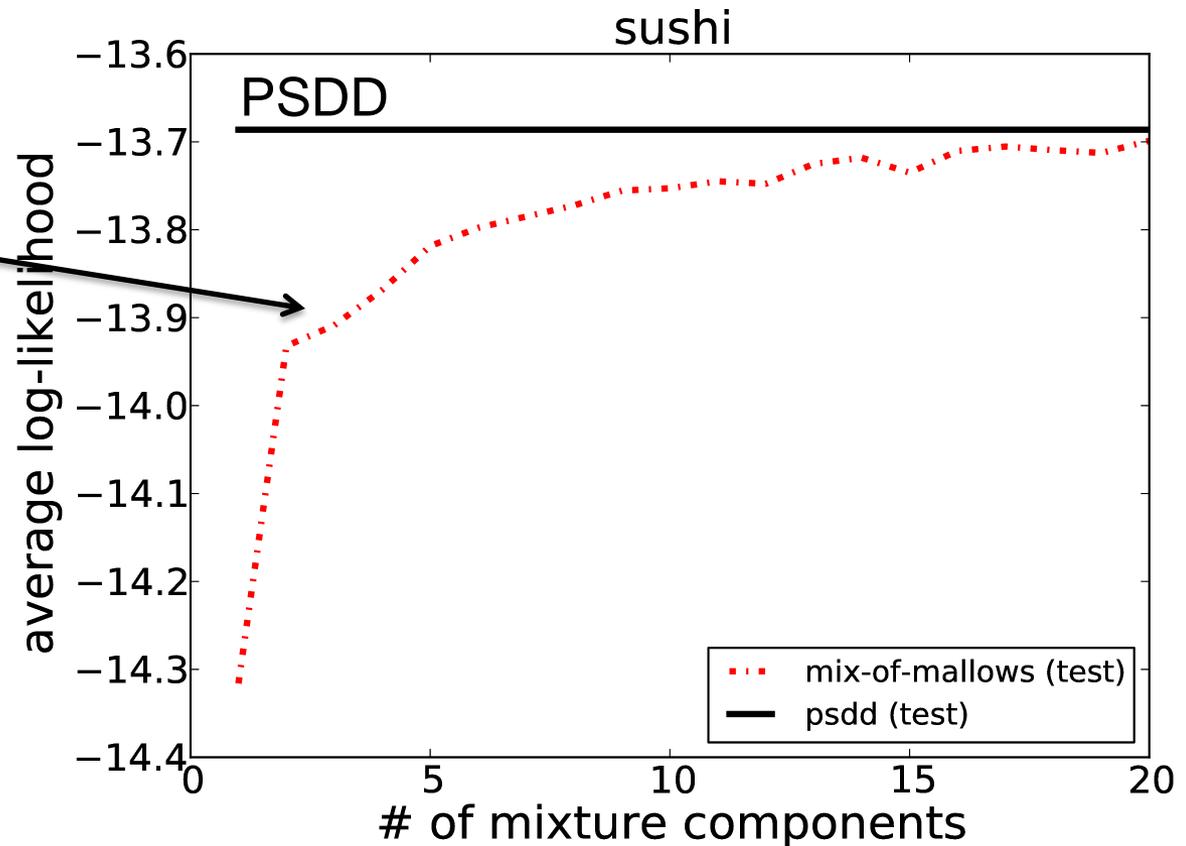
- # of components from 1 to 20
- EM with 10 random seeds
- implementation of Lu & Boutilier



Learning Preference Distributions

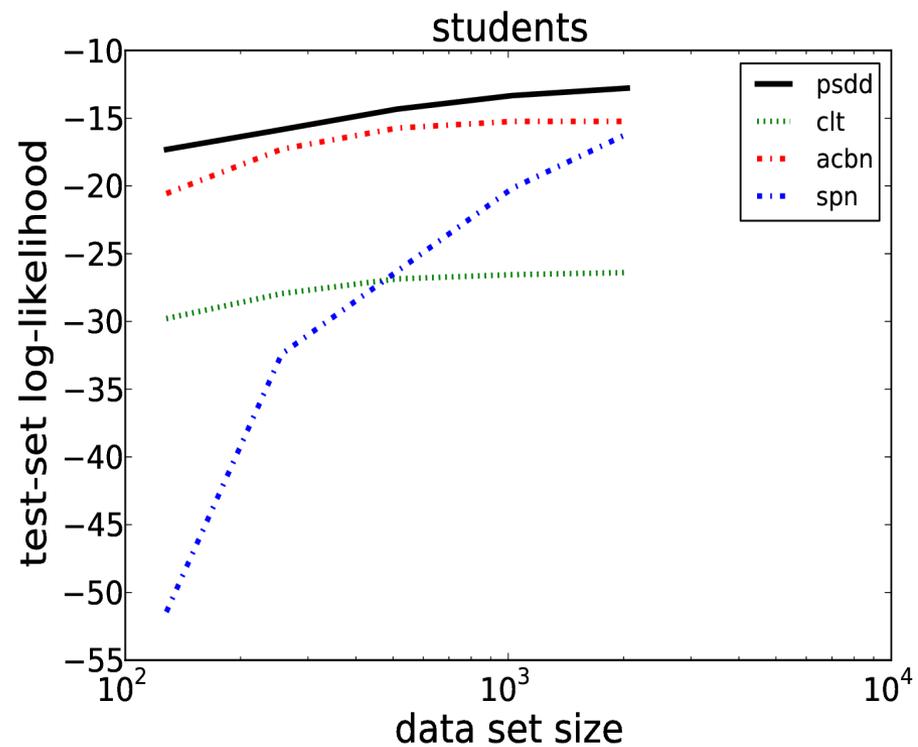
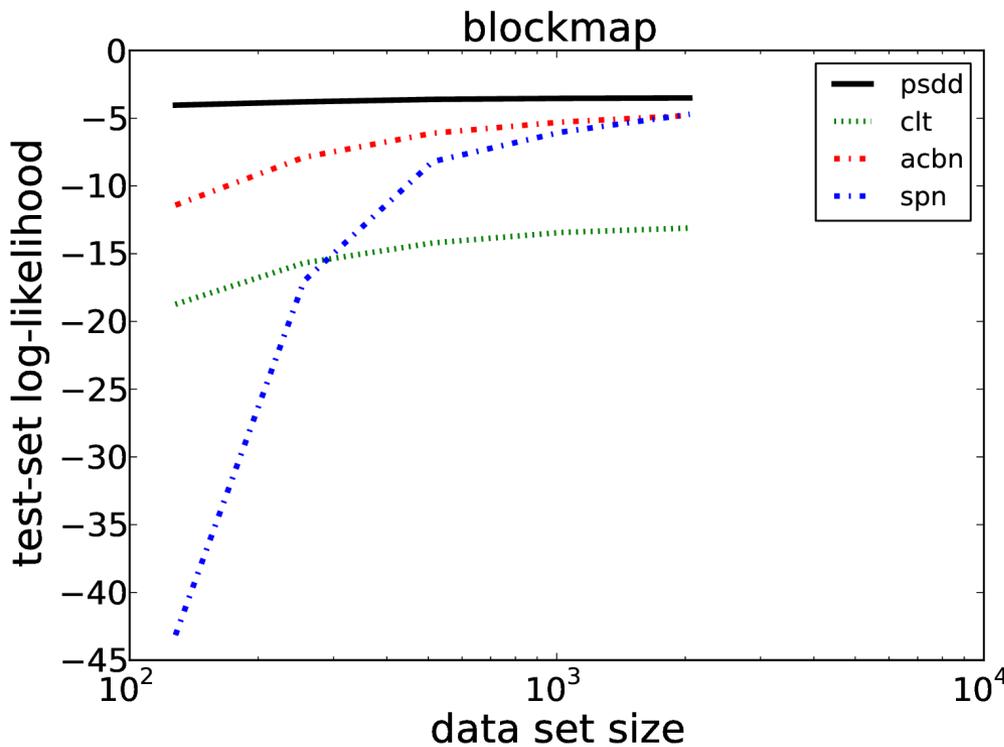
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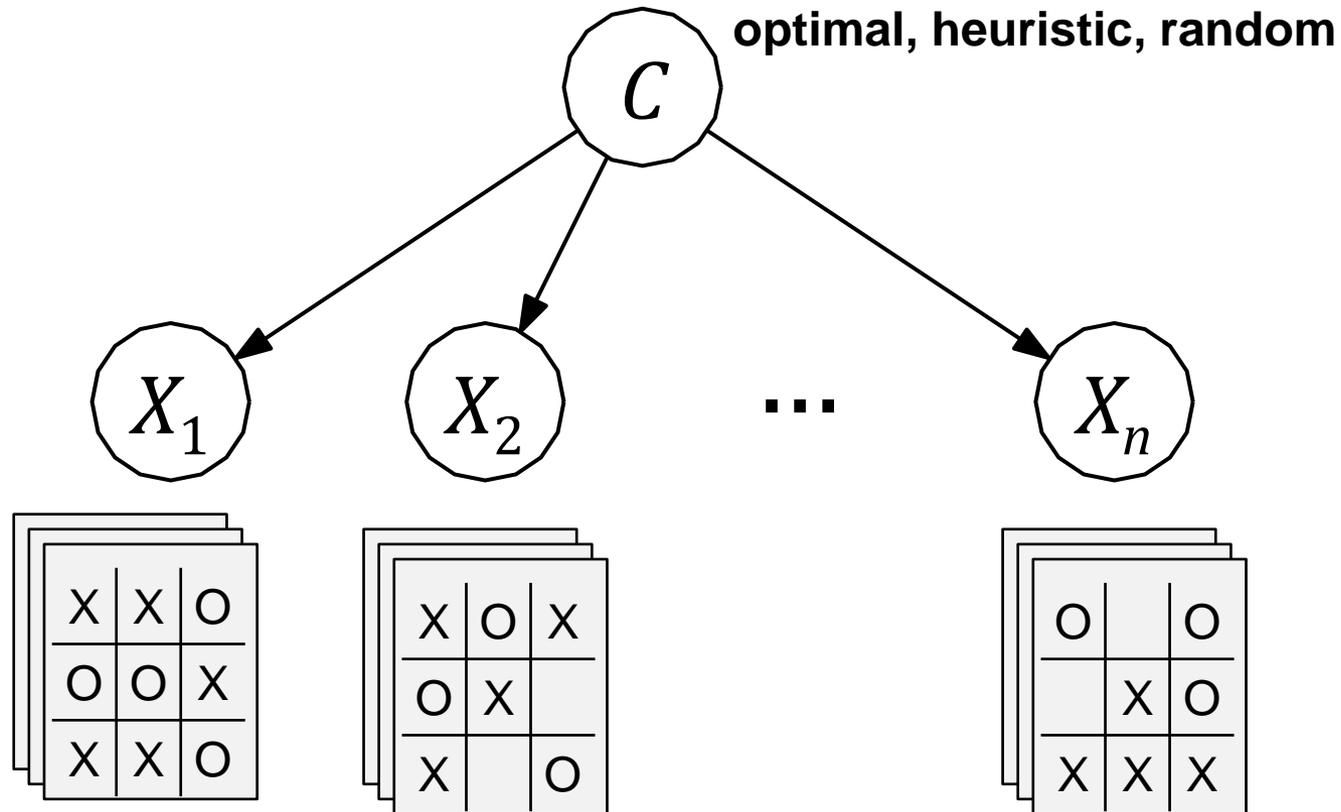


This is the naive approach, without real structure learning!

What happens if you ignore constraints?

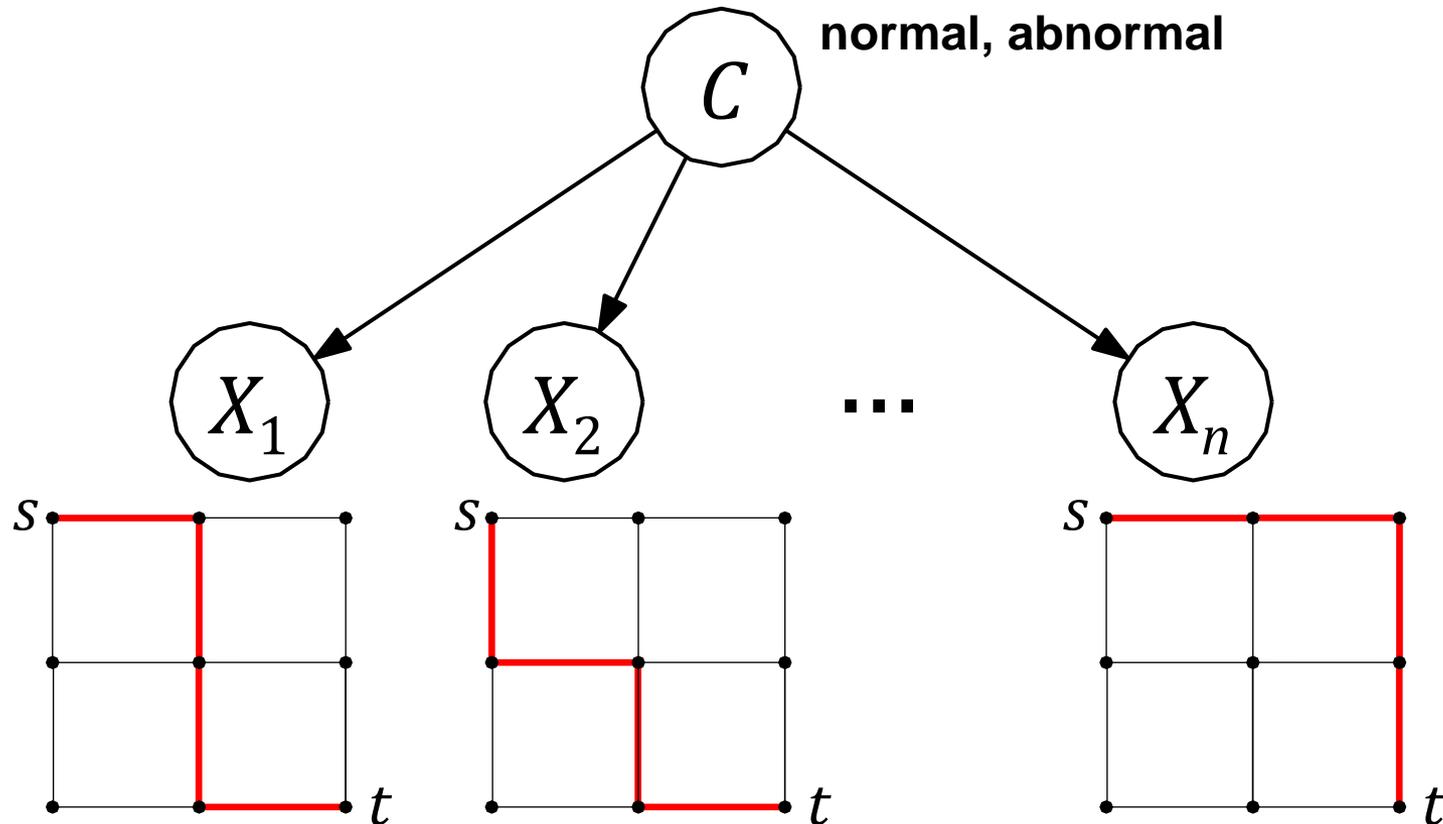


Structured Naïve Bayes Classifier



Attribute with 362,880 values (possible game traces)

Structured Naïve Bayes Classifier



Attribute with 789,360,053,252 values (routes in 8×8 grid)
Ongoing work: learn anomalies from Uber data

Structured datasets and queries

Incomplete Data

a classical
complete dataset

id	X	Y	Z
1	x_1	y_2	z_1
2	x_2	y_1	z_2
3	x_2	y_1	z_2
4	x_1	y_1	z_1
5	x_1	y_2	z_2

closed-form
(maximum-likelihood
estimates are unique)

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closed-form
(maximum-likelihood
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a classical
incomplete dataset

id	X	Y	Z
1	x_1	y_2	?
2	x_2	y_1	?
3	?	?	z_2
4	?	y_1	z_1
5	x_1	y_2	z_2

EM algorithm
(on PSDDs)

Incomplete Data

a classical
complete dataset

id	X	Y	Z
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1	x_1	y_2	?
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3	?	?	z_2
4	?	y_1	z_1
5	x_1	y_2	z_2

EM algorithm
(on PSDDs)

a new type of
incomplete dataset

id	X	Y	Z
1	$X \equiv Z$		
2	x_2 and (y_2 or z_2)		
3	$x_2 \Rightarrow y_1$		
4	$X \oplus Y \oplus Z \equiv 1$		
5	x_1 and y_2 and z_2		

Missed in the
ML literature

Structured Datasets

a classical **complete** dataset
(e.g., total rankings)

id	1 st sushi	2 nd sushi	3 rd sushi	...
1	fatty tuna	sea urchin	salmon roe	...
2	fatty tuna	tuna	shrimp	...
3	tuna	tuna roll	sea eel	...
4	fatty tuna	salmon roe	tuna	...
5	egg	squid	shrimp	...

a classical **incomplete** dataset
(e.g., top- k rankings)

id	1 st sushi	2 nd sushi	3 rd sushi	...
1	fatty tuna	sea urchin	?	...
2	fatty tuna	?	?	...
3	tuna	tuna roll	?	...
4	fatty tuna	salmon roe	?	...
5	egg	?	?	...

Structured Datasets

a classical **complete** dataset
(e.g., total rankings)

id	1 st sushi	2 nd sushi	3 rd sushi	...
1	fatty tuna	sea urchin	salmon roe	...
2	fatty tuna	tuna	shrimp	...
3	tuna	tuna roll	sea eel	...
4	fatty tuna	salmon roe	tuna	...
5	egg	squid	shrimp	...

a new type of **incomplete** dataset
(e.g., **partial** rankings)

id	1 st sushi	2 nd sushi	3 rd sushi	...
1	(fatty tuna > sea urchin) and (tuna > sea eel)			...
2	(fatty tuna is 1 st) and (salmon roe > egg)			...
3	tuna > squid			...
4	egg is last			...
5	egg > squid > shrimp			...

(represents constraints on
possible *total rankings*)

Learning from Incomplete Data

- **Movielens Dataset:**
 - 3,900 movies, 6,040 users, 1m ratings
 - take ratings from 64 most rated movies
 - ratings 1-5 converted to pairwise prefs.
- **PSDD for *partial* rankings**
 - 4 tiers
 - 18,711 parameters

movies by expected tier

rank	movie
1	The Godfather
2	The Usual Suspects
3	Casablanca
4	The Shawshank Redemption
5	Schindler's List
6	One Flew Over the Cuckoo's Nest
7	The Godfather: Part II
8	Monty Python and the Holy Grail
9	Raiders of the Lost Ark
10	Star Wars IV: A New Hope

PSDD Sizes

items n	tier size k	Size		
		SDD	Structured Space	Unstructured Space
8	2	443	840	$1.84 \cdot 10^{19}$
27	3	4,114	$1.18 \cdot 10^9$	$2.82 \cdot 10^{219}$
64	4	23,497	$3.56 \cdot 10^{18}$	$1.04 \cdot 10^{1233}$
125	5	94,616	$3.45 \cdot 10^{31}$	$3.92 \cdot 10^{4703}$
216	6	297,295	$1.57 \cdot 10^{48}$	$7.16 \cdot 10^{14044}$
343	7	781,918	$4.57 \cdot 10^{68}$	$7.55 \cdot 10^{35415}$

Structured Queries

rank	movie
1	Star Wars V: The Empire Strikes Back
2	Star Wars IV: A New Hope
3	The Godfather
4	The Shawshank Redemption
5	The Usual Suspects

Structured Queries

- no other Star Wars movie in top-5
- at least one **comedy** in top-5

rank	movie
1	Star Wars V: The Empire Strikes Back
2	Star Wars IV: A New Hope
3	The Godfather
4	The Shawshank Redemption
5	The Usual Suspects

Structured Queries

- no other Star Wars movie in top-5
- at least one **comedy** in top-5

rank	movie
1	Star Wars V: The Empire Strikes Back
2	Star Wars IV: A New Hope
3	The Godfather
4	The Shawshank Redemption
5	The Usual Suspects

rank	movie
1	Star Wars V: The Empire Strikes Back
2	American Beauty
3	The Godfather
4	The Usual Suspects
5	The Shawshank Redemption

Structured Queries

- no other Star Wars movie in top-5
- at least one **comedy** in top-5

rank	movie
1	Star Wars V: The Empire Strikes Back
2	Star Wars IV: A New Hope
3	The Godfather
4	The Shawshank Redemption
5	The Usual Suspects

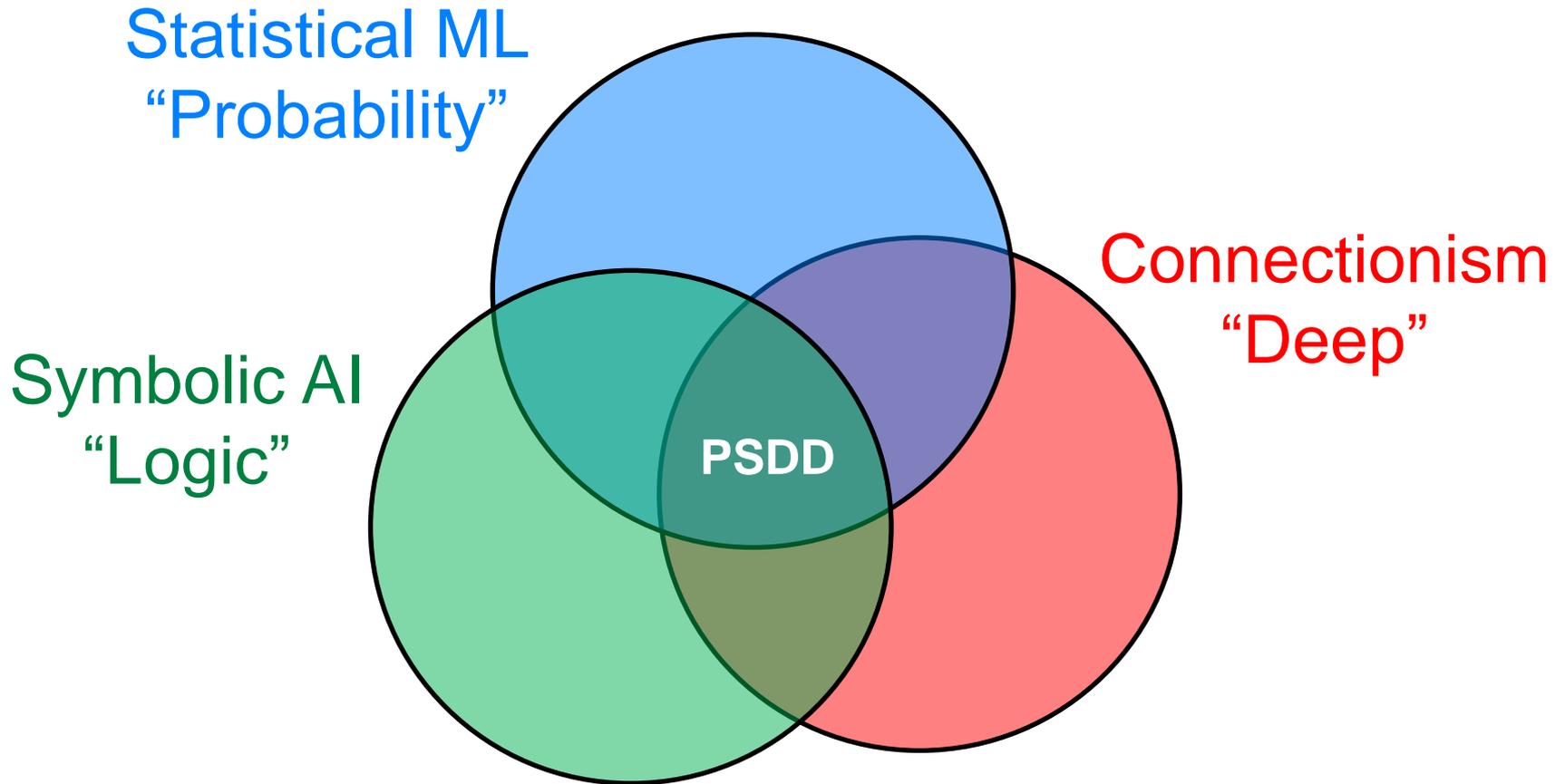
rank	movie
1	Star Wars V: The Empire Strikes Back
2	American Beauty
3	The Godfather
4	The Usual Suspects
5	The Shawshank Redemption

diversified recommendations via
logical constraints

Conclusions

- Structured spaces are everywhere 😊
- Roles of Boolean constraints in ML
 - Domain constraints and combinatorial objects (**structured probability space**)
 - Incomplete examples (**structured datasets**)
 - Questions and evidence (**structured queries**)
- Learn distributions over combinatorial objects
- Strong properties for inference and learning:
Probabilistic sentential decision diagram (PSDD)

Conclusions



References

Probabilistic Sentential Decision Diagrams

Doga Kisa, Guy Van den Broeck, Arthur Choi and Adnan Darwiche

KR, 2014

Learning with Massive Logical Constraints

Doga Kisa, Guy Van den Broeck, Arthur Choi and Adnan Darwiche

ICML 2014 workshop

Tractable Learning for Structured Probability Spaces

Arthur Choi, Guy Van den Broeck and Adnan Darwiche

IJCAI, 2015

Tractable Learning for Complex Probability Queries

Jessa Bekker, Jesse Davis, Arthur Choi, Adnan Darwiche, Guy Van den Broeck.

NIPS, 2015

Structured Features in Naive Bayes Classifiers

Arthur Choi, Nazgol Tavabi and Adnan Darwiche

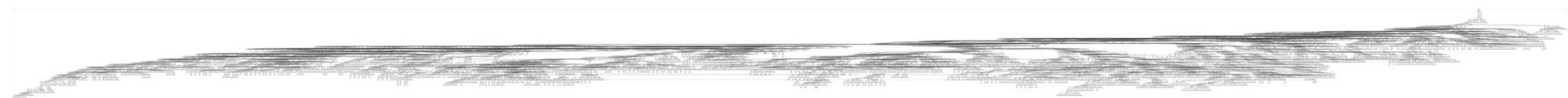
AAAI, 2016

Tractable Operations on Arithmetic Circuits

Jason Shen, Arthur Choi and Adnan Darwiche

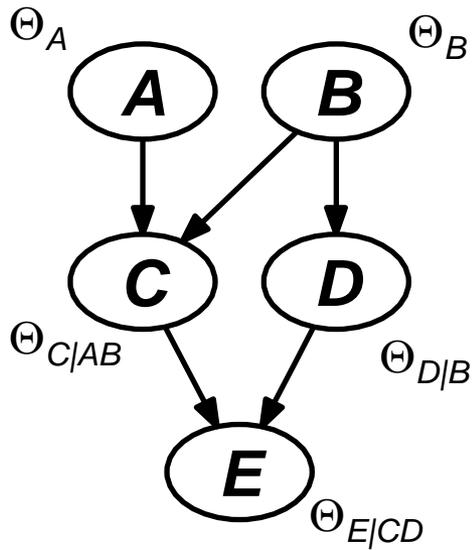
NIPS, 2016

Questions?



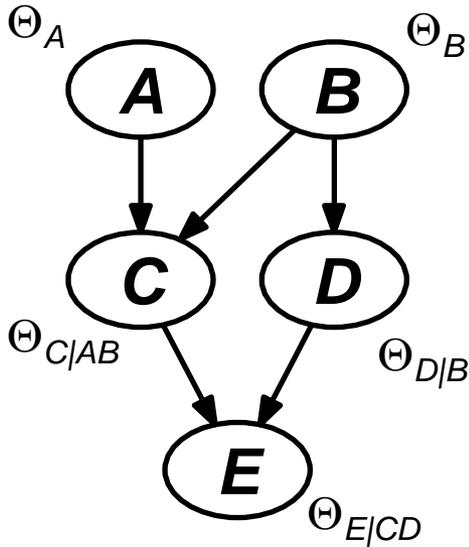
PSDD with 15,000 nodes

Compiling PGMs into PSDDs



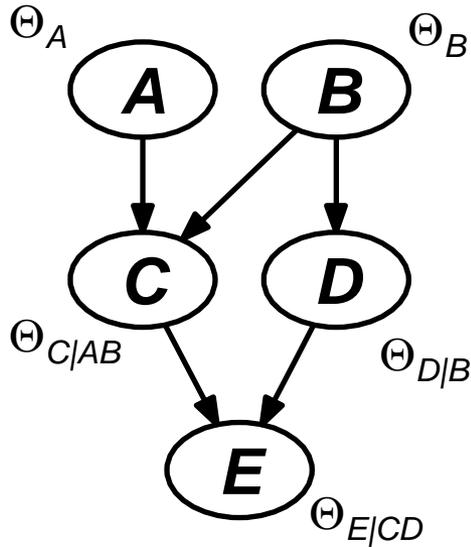
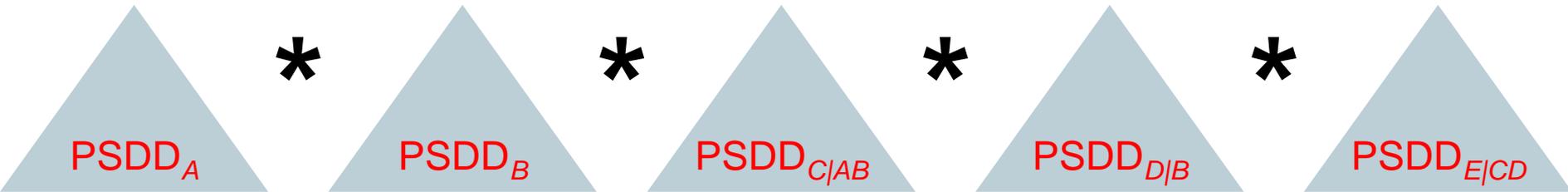
Compiling PGMs into PSDDs

$$\Pr(A,B,C,D,E) = \Theta_A \Theta_B \Theta_{C|AB} \Theta_{D|B} \Theta_{E|CD}$$



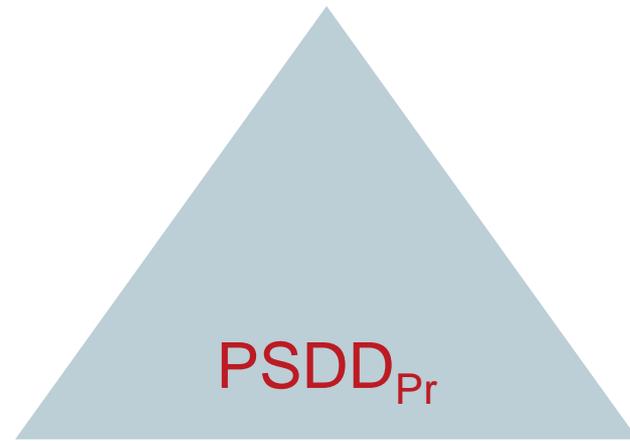
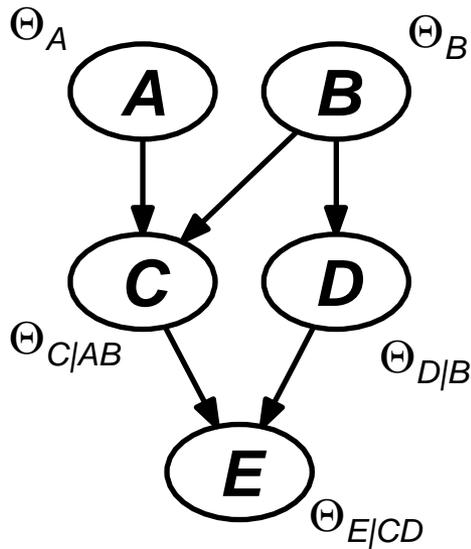
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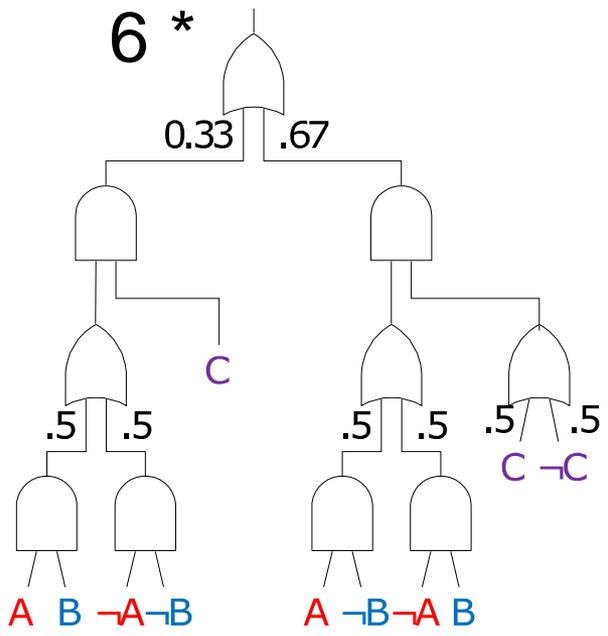


Compiling PGMs into PSDDs

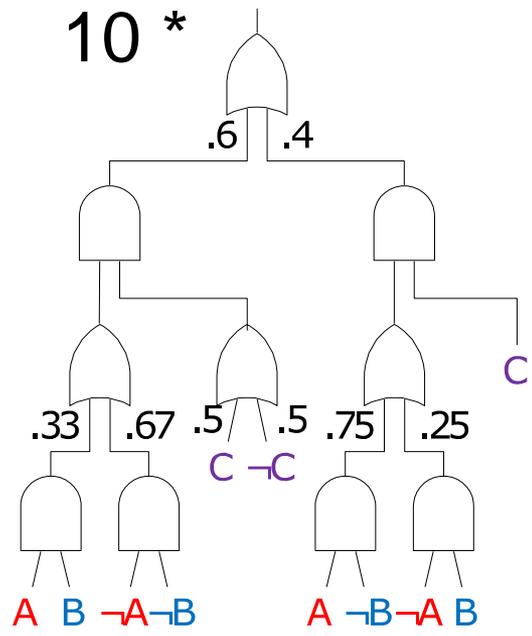
$$\Pr(A,B,C,D,E) = \Theta_A \Theta_B \Theta_{C|AB} \Theta_{D|B} \Theta_{E|CD}$$



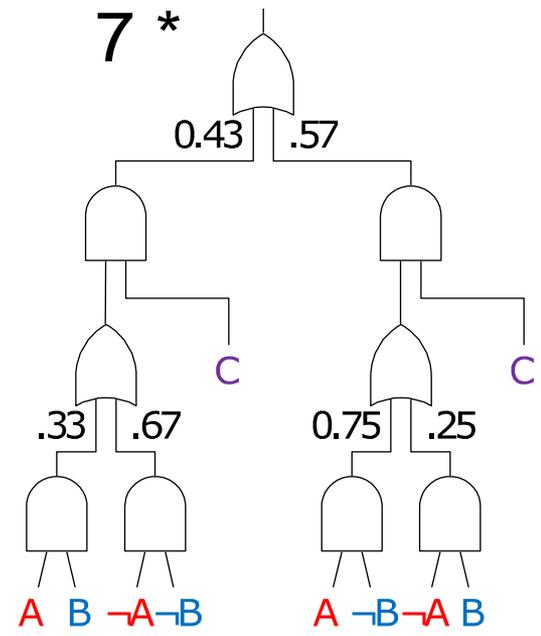
Sparse tables [Larkin & Dechter 2003], ADDs [Bahar, et al. 1993], AOMDDs [Mateescu, et al., 2008], PDGs [Jaeger, 2004]



*



=



A	B	C	f
T	T	T	1
T	T	F	0
T	F	T	1
T	F	F	1
F	T	T	1
F	T	F	1
F	F	T	1
F	F	F	0

*

A	B	C	g
T	T	T	1
T	T	F	1
T	F	T	3
T	F	F	0
F	T	T	1
F	T	F	0
F	F	T	2
F	F	F	2

=

A	B	C	f*g
T	T	T	1
T	T	F	0
T	F	T	3
T	F	F	0
F	T	T	1
F	T	F	0
F	F	T	2
F	F	F	0