



# Data and Knowledge in Neuro-Symbolic Learning

Guy Van den Broeck

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Pure (Logic) Reasoning Pure Learning



**Pure Learning** 

- Slow thinking: deliberative, cognitive, model-based, extrapolation
- Amazing achievements until this day
- "Pure logic is brittle" noise, uncertainty, incomplete knowledge, ...



**Pure (Logic) Reasoning** 

**Pure Learning** 

- Fast thinking: instinctive, perceptive, model-free, interpolation
- Amazing achievements recently
- "Pure learning is brittle"

bias, algorithmic fairness, interpretability, explainability, adversarial attacks, unknown unknowns, calibration, verification, missing features, missing labels, data efficiency, shift in distribution, general robustness and safety fails to incorporate a sensible model of the world





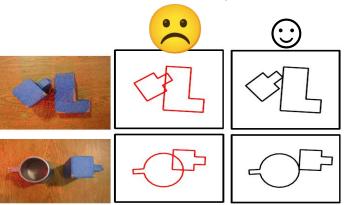
- Learn statistical models subject to symbolic knowledge
- Integrate reasoning into modern learning algorithms

Today: Deep learning with structured output constraints Learning monotonic neural networks

### Knowledge in Vision, Robotics, NLP



People appear at most once in a frame



Rigid objects don't overlap

At least one verb in each sentence.

If X and Y are married, then they are people.

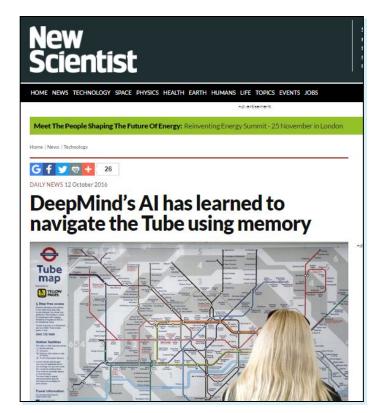
#### Activity Recognition & Task Guidance

Cut the orange before squeezing the orange

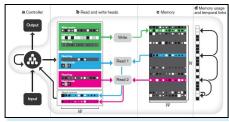




#### Motivation: Deep Learning



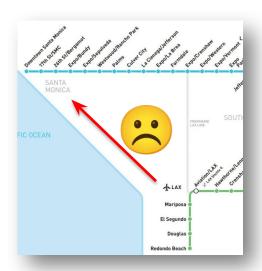




[Graves, A., Wayne, G., Reynolds, M., Harley, T., Danihelka, I., Grabska-Barwińska, A., et al.. (2016). Hybrid computing using a neural network with dynamic external memory. *Nature*, *538*(7626), 471-476.]

#### Motivation: Deep Learning

DeepMind's latest technique uses external memory to solve tasks that require logic and reasoning — a step toward more human-like Al.



#### ... but ...

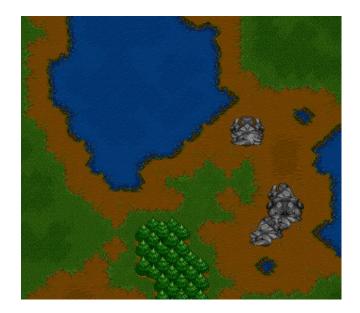
optimal planner recalculating a shortest path to the end node. To ensure that the network always moved to a valid node, the output distribution was renormalized over the set of possible triples outgoing from the current node. The performance

it also received input triples during the answer phase, indicating the actions chosen on the previous time-step. This makes the problem a 'structured prediction'

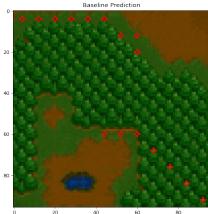
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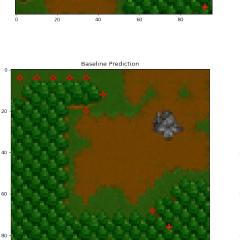
#### Warcraft Shortest Path

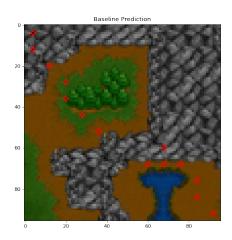
Predicting the minimum-cost path

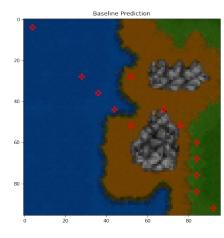




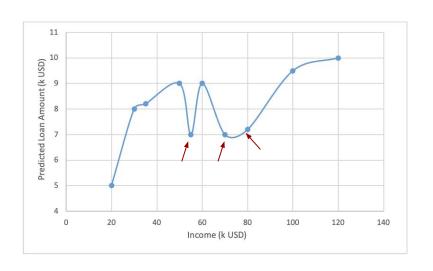








#### **Predict Loan Amount**





Neural Network Model: Increasing income can decrease the approved loan amount

Monotonicity (Prior Knowledge):

Increasing income should increase the approved loan amount

# Knowledge vs. Data

- Where did the world knowledge go?
  - Python scripts
    - Decode/encode cleverly
    - Fix inconsistent beliefs
  - Rule-based decision systems
  - Dataset design
  - "a big hack" (with author's permission)

# Knowledge vs. Data

- Where did the world knowledge go?
  - Python scripts
    - Decode/encode cleverly
    - Fix inconsistent beliefs
  - Rule-based decision systems
  - Dataset design
  - "a big hack" (with author's permission)
- In some sense we went backwards

Less principled, scientific, and intellectually satisfying ways of incorporating knowledge

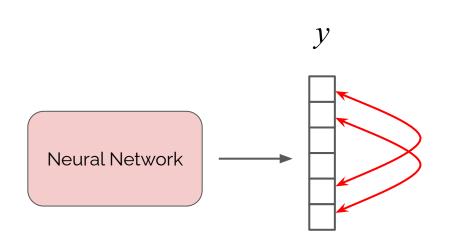
# Deep Learning with Constraints

#### A PyTorch Framework for Learning with Constraints

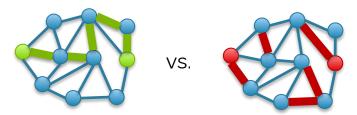
Kareem Ahmed Tao Li Thy Ton Quan Guo, Kai-Wei Chang Parisa Kordjamshidi Vivek Srikumar Guy Van den Broeck Sameer Singh

http://pylon-lib.github.io

#### Declarative Knowledge of the Output



How is the output structured? Are all possible outputs valid?



How are the outputs related to each other?

Learning this from data is inefficient Much easier to express this declaratively

How can do we inject declarative knowledge into PyTorch training code?

Library that extends PyTorch to allow injection of declarative knowledge

- Easy to Express Knowledge: users write arbitrary constraints on the output
- Integrates with PyTorch: minimal change to existing code
- Efficient Training: compiles into loss that can be efficiently optimized
  - Exact semantic loss
  - Monte-carlo estimate of loss
  - T-norm approximation
  - o your solver?

```
PyTorch Code

for i in range(train_iters):
    ...
    py = model(x)
    ...
    loss = CrossEntropy(py,...)
```

1) Specify knowledge as a predicate

```
def check(y):
    ...
    return isValid
```

```
PyTorch Code

for i in range(train_iters):
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    loss += constraint_loss(check)(py)
```

1) Specify knowledge as a predicate

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def check(y):
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2 Add as loss to training

```
loss += constraint_loss(check)
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PyTorch Code

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1 Specify knowledge as a predicate

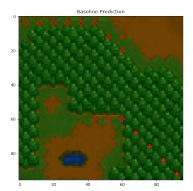
```
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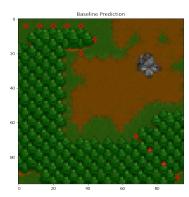
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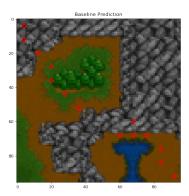
3 pylon derives the gradients (solves a combinatorial problem)

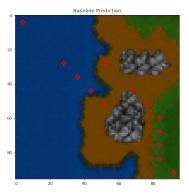
#### without constraint



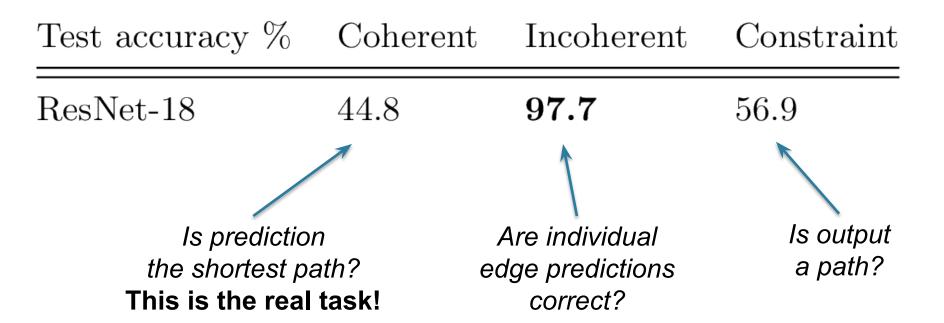


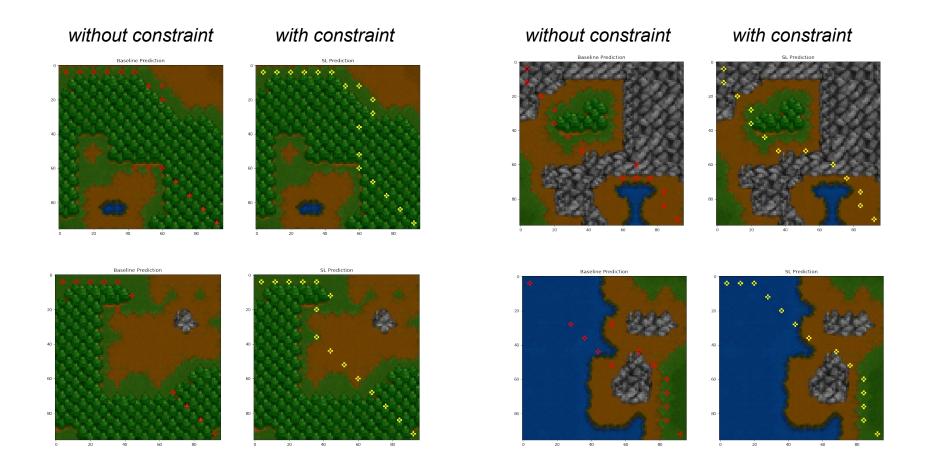
#### without constraint





#### Warcraft min-cost simple-path prediction results





#### Warcraft min-cost simple-path prediction results

Test accuracy %	Coherent	Incoherent	Constraint 56.9	
ResNet-18	44.8	97.7		
+ Semantic loss	50.9	97.7	67.4	

### **Semantic Loss**

 $\underline{\mathbf{Q}}$ : How close is output  $\boldsymbol{p}$  to satisfying constraint  $\alpha$ ?

<u>A</u>: Semantic loss function  $L(\alpha, \mathbf{p})$ 

- Axioms, for example:
  - If  $\alpha$  constrains to one label,  $L(\alpha, \mathbf{p})$  is cross-entropy
  - If α implies β then  $L(\alpha, \mathbf{p}) \ge L(\beta, \mathbf{p})$  (α more strict)

- Implied Properties:
  - If α is equivalent to β then  $L(\alpha, \mathbf{p}) = L(\beta, \mathbf{p})$  Loss!
  - If **p** is Boolean and satisfies  $\alpha$  then  $L(\alpha, \mathbf{p}) = 0$

### Axioms imply unique semantic loss:

$$L^{s}(\alpha, p) \propto -\log \sum_{\mathbf{x} \models \alpha} \prod_{i: \mathbf{x} \models X_{i}} p_{i} \prod_{i: \mathbf{x} \models \neg X_{i}} (1 - p_{i})$$

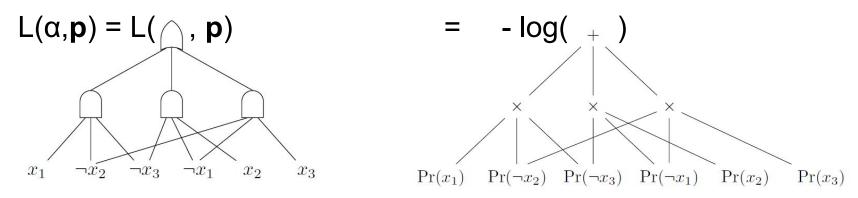
Probability of satisfying constraint α after sampling from neural net output layer **p** 

In general: #P-hard 😕

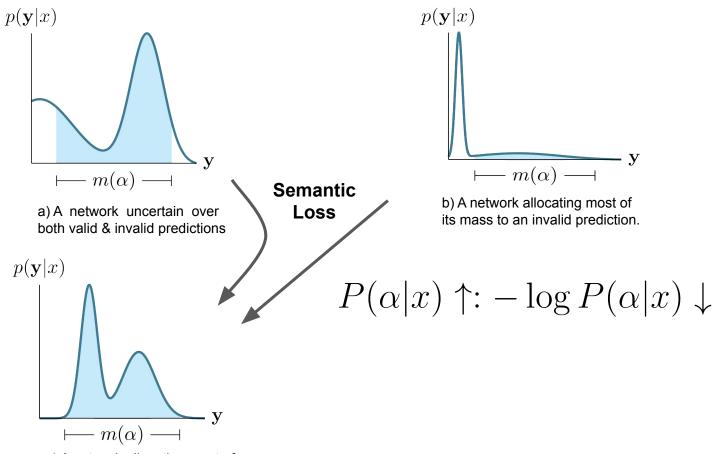
We do this probabilistic-logical reasoning during learning in a computation graph

## Logical Computation Graphs

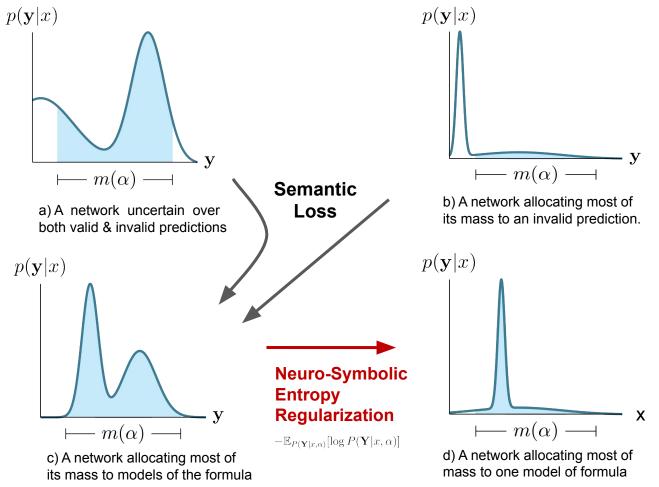
- Logical circuits that can count solutions (#SAT)
- Also compute semantic loss efficiently in size of circuit



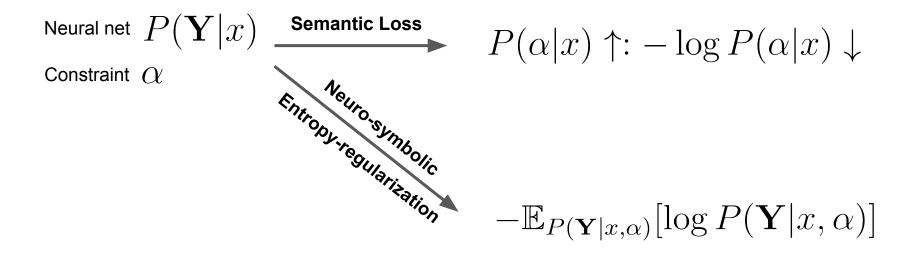
- Compilation into circuit by SAT solvers (once)
- Add circuit to neural network output in pytorch/tensorflow/...



c) A network allocating most of its mass to models of the formula



# Two complementary neuro-symbolic losses



#### Warcraft min-cost simple-path prediction results

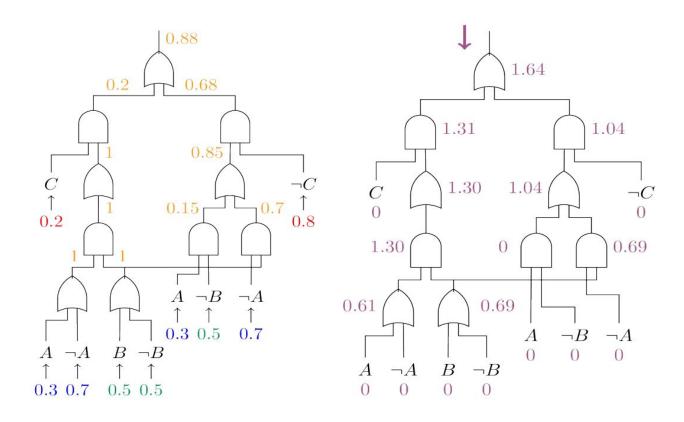
Test accuracy %	Coherent	Incoherent	Constraint	
ResNet-18	44.8	97.7	56.9	
Semantic loss	50.9	97.7	67.4	
+ Entropy All	51.5	97.6	67.7	
+ Entropy Circuit	55.0	$\boldsymbol{97.9}$	69.8	

#### Joint entity-relation extraction in natural language processing

# Labels		3	5	10	15	25	50	75
ACE05	Baseline Self-training Product t-norm	$\begin{array}{c} 4.92 \pm 1.12 \\ 7.72 \pm 1.21 \\ 8.89 \pm 5.09 \end{array}$	$  \begin{array}{c} 7.24 \pm 1.75 \\ 12.83 \pm 2.97 \\ 14.52 \pm 2.13 \end{array} $	$16.22 \pm 3.08$	$ \begin{vmatrix} 15.07 \pm 1.79 \\ 17.55 \pm 1.41 \\ 21.80 \pm 7.67 \end{vmatrix} $		$ \begin{vmatrix} 28.96 \pm 0.98 \\ 32.90 \pm 1.71 \\ 34.12 \pm 2.75 \end{vmatrix}$	$37.15 \pm 1.42$
	Semantic Loss + Entropy All + Entropy Circuit	$12.00 \pm 3.81$ $14.80 \pm 3.70$ $14.72 \pm 1.57$	$15.78 \pm 1.90$	$23.34 \pm 4.07$	$ \begin{vmatrix} 27.35 \pm 3.10 \\ 28.09 \pm 1.46 \\ \textbf{31.17} \pm \textbf{1.68} \end{vmatrix} $	$31.13 \pm 2.26$		$38.49 \pm 1.74$ $39.39 \pm 1.21$ $41.28 \pm 0.46$
SciERC	Baseline Self-training Product t-norm	$2.71 \pm 1.1$ $3.56 \pm 1.4$ $6.50 \pm 2.0$	$2.94 \pm 1.0$ $3.04 \pm 0.9$ $8.86 \pm 1.2$	$\begin{array}{c} 3.49 \pm 1.8 \\ 4.14 \pm 2.6 \\ 10.92 \pm 1.6 \end{array}$	$ \begin{vmatrix} 3.56 \pm 1.1 \\ 3.73 \pm 1.1 \\ 13.38 \pm 0.7 \end{vmatrix} $	$8.83 \pm 1.0$ $9.44 \pm 3.8$ $13.83 \pm 2.9$	$ \begin{vmatrix} 12.32 \pm 3.0 \\ 14.82 \pm 1.2 \\ 19.20 \pm 1.7 \end{vmatrix} $	$\begin{array}{ c c }\hline 12.49 \pm 2.6 \\ 13.79 \pm 3.9 \\ 19.54 \pm 1.7\end{array}$
	Semantic Loss + Entropy All + Entropy Circuit	$6.47 \pm 1.02$ $6.26 \pm 1.21$ $6.19 \pm 2.40$	$\begin{array}{c} \textbf{9.31} \pm \textbf{0.76} \\ 8.49 \pm 0.85 \\ 8.11 \pm 3.66 \end{array}$	$11.12 \pm 1.22$	$ \begin{vmatrix} 12.97 \pm 2.86 \\ 14.10 \pm 2.79 \\ \textbf{15.47} \pm \textbf{2.19} \end{vmatrix} $	$17.25 \pm 2.75$	$\begin{array}{c} 20.47 \pm 2.50 \\ \textbf{22.42} \pm \textbf{0.43} \\ 22.14 \pm 1.46 \end{array}$	$24.37 \pm 1.62$

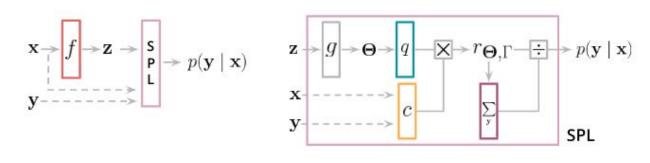
Table 5: Experimental results for joint entity-relation extraction on ACE05 and SciERC. #Labels indicates the number of labeled data points made available to the network per relation. The remaining training set is stripped of labels and is utilized in an unsupervised manner: enforce the constraint or minimize the entropy. We report averages and errors across 3 different runs.

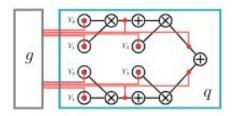
#### Probabilistic-Logical Reasoning using Circuits



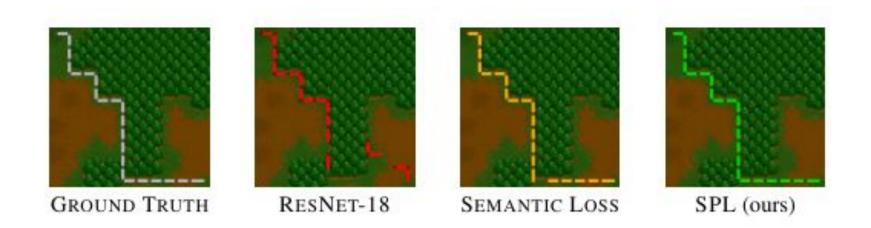
#### Semantic Probabilistic Layers

- How to give a 100% guarantee that Boolean constraints will be satisfied?
- Bake the constraint into the neural network as a special layer





#### Warcraft Shortest Path



#### Hierarchical Multi-Label Classification

"if the image is classified as a dog, it must also be classified as an animal"

"if the image is classified as an animal, it must be classified as either cat or dog"

DATASET	EXACT MATCH				
	HMCNN	MLP+SPL	MLP+SPL++		
CELLCYCLE	3.04	4.14	4.29		
DERISI	1.65	2.51	2.99		
EISEN	5.38	6.56	7.17		
EXPR	4.18	6.12	6.13		
GASCH1	3.66	5.37	5.54		
GASCH2	3.02	4.49	4.58		
SEQ	5.15	8.36	8.48		
SPO	2.05	2.29	3.09		
DIATOMS	48.48	59.11	57.69		
ENRON	6.06	9.54	9.55		
IMCLEF07A	79.52	85.68	85.88		
IMCLEF07D	76.04	83.20	83.10		

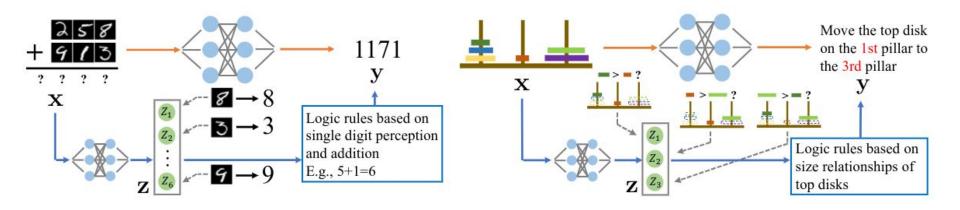
## Neuro-Symbolic Learning Settings

#### Learn

- 1. neural network given symbols and constraints and data
- 2. neural network and constraints given symbols and data
- 3. neural network and constraints and symbols given data

Everyone is working on 1. Ongoing work on 2.

# Neuro-Symbolic Joint Training



Learn invariant features using neural networks. Learn logic to tie it all together.

# Neuro-Symbolic Joint Training

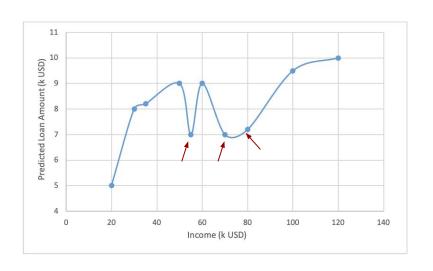
Multi-digit addition [test seq length + train/test img]					Tower of Hanoi			
w/ test	10 w/ test	20 w/ test	5 w/ train	10 w/ train	20 w/ train	Task #1	Task #2	Task #3
88.30	77.46	timeout	94.92	89.74	timeout	89.28	97.96	89.33
81.40	56.97	39.05	88.92	77.40	63.23	78.26	98.32	74.36
81.49	59.64	33.83 63.55	81.88 <b>89.97</b>	59.96 <b>86.07</b>	37.85 <b>71.96</b>	76.20 <b>85.16</b>	97.87 97.94	73.87 <b>85.49</b>
(X) (X)	w/ test 38.30 31.40	w/ test 10 w/ test 38.30 77.46 31.40 56.97 31.49 59.64	w/ test 10 w/ test 20 w/ test 38.30 77.46 timeout 31.40 56.97 39.05 31.49 59.64 33.83	w/ test 10 w/ test 20 w/ test 5 w/ train 38.30 77.46 timeout 94.92 31.40 56.97 39.05 88.92 31.49 59.64 33.83 81.88	w/ test 10 w/ test 20 w/ test 5 w/ train 10 w/ train 38.30 77.46 timeout 94.92 89.74 31.40 56.97 39.05 88.92 77.40 31.49 59.64 33.83 81.88 59.96	w/ test 10 w/ test 20 w/ test 5 w/ train 10 w/ train 20 w/ train 38.30 77.46 timeout 94.92 89.74 timeout 31.40 56.97 39.05 88.92 77.40 63.23 31.49 59.64 33.83 81.88 59.96 37.85	w/ test 10 w/ test 20 w/ test 5 w/ train 10 w/ train 20 w/ train Task #1  88.30 77.46 timeout 94.92 89.74 timeout 89.28  81.40 56.97 39.05 88.92 77.40 63.23 78.26  81.49 59.64 33.83 81.88 59.96 37.85 76.20	w/ test 10 w/ test 20 w/ test 5 w/ train 10 w/ train 20 w/ train Task #1 Task #2 38.30 77.46 timeout 94.92 89.74 timeout 89.28 97.96 31.40 56.97 39.05 88.92 77.40 63.23 78.26 <b>98.32</b> 31.49 59.64 33.83 81.88 59.96 37.85 76.20 97.87

Learn invariant features using neural networks. Learn logic to tie it all together.

# Neural Networks

Monotonicity Invariants for

#### **Predict Loan Amount**



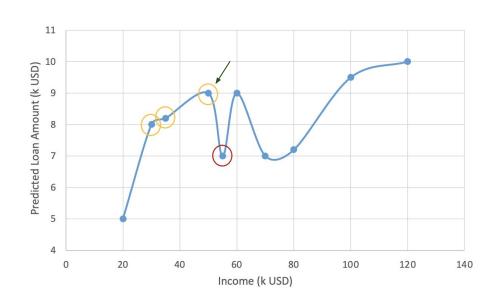


Neural Network Model: Increasing income can decrease the approved loan amount

Monotonicity (Prior Knowledge):

Increasing income should increase the approved loan amount

### Counterexamples

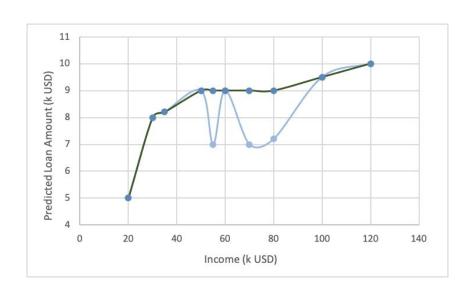


$$\exists x, y \ x \le y \implies f(x) > f(y)$$

Computed using SMT(LRA) logical reasoning solver

Maximal counterexamples (largest violation) using OMT

## Counterexample-Guided Predictions



#### **Monotonic Envelope:**

- Replace each prediction by its maximal counterexample
- Envelope construction is online (during prediction)
- Guarantees monotonic predictions for any ReLU neural net
- Works for high-dimensional input
- Works for multiple monotonic features

### Monotonic Envelope: Performance

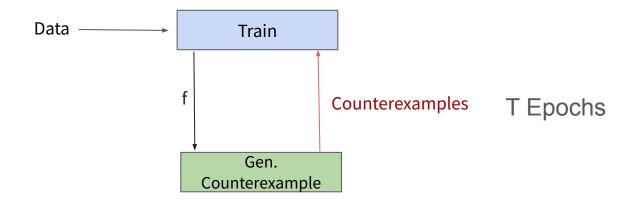
Dataset	Feature	NN <sub>b</sub>	Envelope	
Auto-MPG	weight Displ. W,D W,D,HP		9.19±3.41 9.63±2.61 9.63±2.61 9.63±2.61	
Boston	Rooms Crime	14.37±2.4 14.37±2.4	14.19±2.28 14.02±2.17	

Dataset	<b>Feature</b>	$NN_b$	Envelope	
Heart	Trestbps Chol. T,C	$0.85\pm0.04$ $0.85\pm0.04$ $0.85\pm0.04$	$0.85\pm0.04 \\ 0.85\pm0.05 \\ 0.85\pm0.05$	
Adult	Cap. Gain Hours	0.84 0.84	0.84 0.84	

Guaranteed monotonicity at little to no cost

## Counterexample-Guided Learning

How to use monotonicity to improve model quality? "Monotonicity as inductive bias"



## Counterexample-Guided Learning: Performance

O <del>.</del>							
Dataset	Feature	NN <sub>b</sub>	CGL	Dataset	Feature	NN <sub>b</sub>	CGL
Auto-MPG	Weight Displ. W,D W,D,HP	9.33±3.22 9.33±3.22 9.33±3.22 9.33±3.22	$9.04{\pm}2.76$ $9.08{\pm}2.87$ $8.86{\pm}2.67$ $8.63{\pm}2.21$	Heart	Trestbps Chol. T,C	0.85±0.04 <b>0.85</b> ± <b>0.04</b> 0.85±0.04	$0.86\pm0.02 \\ 0.85\pm0.05 \\ 0.86\pm0.06$
Boston	Rooms Crime	14.37±2.4 14.37±2.4	12.24±2.87 11.66±2.89	Adult	Cap. Gain Hours	0.84 0.84	0.84 0.84

## Monotonicity is a *great* inductive bias for learning

# Counterexample-Guided Monotonicity Enforced Training (COMET)

Table 4: Monotonicity is an effective inductive bias. COMET outperforms Min-Max networks on all datasets. COMET outperforms DLN in regression datasets and achieves similar results in classification datasets.

Dataset	Features	Min-Max	DLN	Сомет
Auto- MPG	Weight Displ. W,D W,D,HP	$9.91\pm1.20$ $11.78\pm2.20$ $11.60\pm0.54$ $10.14\pm1.54$	$16.77\pm2.57$ $16.67\pm2.25$ $16.56\pm2.27$ $13.34\pm2.42$	8.92±2.93 9.11±2.25 8.89±2.29 8.81±1.81
Boston	Rooms Crime	$30.88 \pm 13.78$ $25.89 \pm 2.47$	$15.93 \pm 1.40$ $12.06 \pm 1.44$	11.54±2.55 11.07±2.99

Dataset	Features	Min-Max	DLN	Сомет
Heart	Trestbps Chol. T,C	0.75±0.04 0.75±0.04 0.75±0.04	$0.85\pm0.02$ $0.85\pm0.04$ $0.86\pm0.02$	0.86±0.03 0.87±0.03 0.86±0.03
Adult	Cap. Gain Hours	0.77 0.73	0.84 0.85	<b>0.84</b> 0.84

#### COMET = Provable Guarantees + SotA Results

## The Al Dilemma



- Knowledge is (hidden) everywhere in ML
- A little bit of reasoning goes a long way!

Deep learning with structured output constraints Learning monotonic neural networks

## **Thanks**

This was the work of many wonderful students/postdoc/collaborators!

References: <a href="http://starai.cs.ucla.edu/publications/">http://starai.cs.ucla.edu/publications/</a>