Strudel: Learning Structured-Decomposable Probabilistic Circuits

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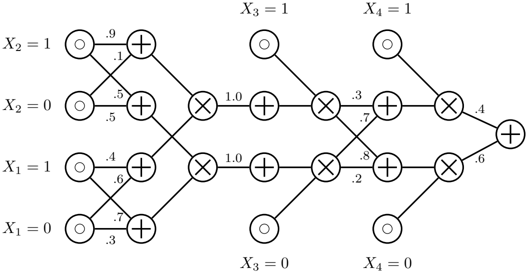
*September 23th, 2020 - **Probabilistic Graphical Models (PGM)***

Learning Structured-Decomposable ***Probabilistic Circuits***

Probabilistic circuits (PCs) are ***tractable probabilistic models***

$\Longrightarrow$ ***exact*** *and* ***efficient*** *inference!*



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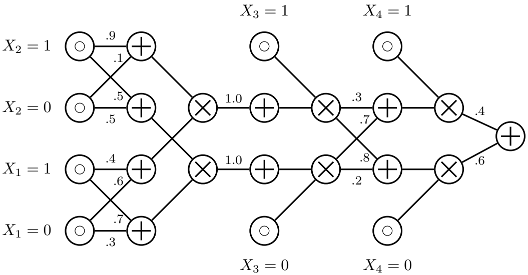

known under many different names: ***SPNs***, ***PSDDs***, ***CNets***,...

$\Longrightarrow$ *checkout Guy's tutorial tomorrow!*

Learning ***Structured-Decomposable*** *Probabilistic Circuits*

To answer different classes of ***tractable inferences***, PCs are required to have certain ***structure properties*** [1]

	EVI	*MAR*	*CON*	*MAP*
smooth		✓	✓	
decomposable		✓	✓	✓
deterministic				✓

[1] Probabilistic circuits tutorial @ ECML-PKDD available at `https://www.youtube.com/watch?v=2RAG5-L9R70`

Learning Structured-Decomposable Probabilistic Circuits

To answer different classes of ***tractable inferences***, PCs are required to have certain ***structure properties*** [1]

	EVI	*MAR*	*CON*	*MAP*	*KLD*	*Multiply*	*Expected Predictions*
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decomposable		✓	✓	✓			
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Learning ***Structured-Decomposable*** *Probabilistic Circuits*

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smooth		✓	✓		✓	✓	✓
decomposable		✓	✓	✓	✓	✓	✓
deterministic				✓	✓		
struct.-decomp.					✓	✓	✓

For advanced queries...we need ***structured-decomposable*** PCs!

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Learning *Structured-Decomposable Probabilistic Circuits*

1. Initial structure
2. Generate candidate improvements
3. Score candidates $\Rightarrow$ *expensive!*
4. go to 2.

LearnPSDD *[Liang et al. 2017]*

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1. Better initial structure
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   $\Longrightarrow$ *fast heuristics!*
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Strudel

Learning *Structured-Decomposable Probabilistic Circuits*

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Strudel

faster single structure learning $\longrightarrow$ fast mixture models!

Outline

Probabilistic Circuits

Scalable Learning Algorithm

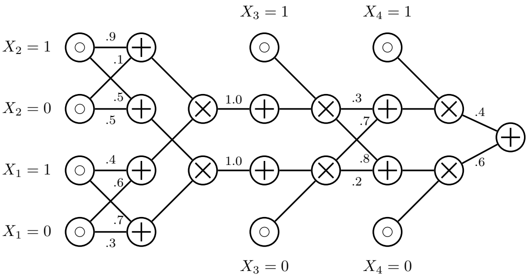
- Structured Decomposability and Learning Algorithm
- Scale More with Mixtures

Advanced Queries

Conclusions

Probabilistic Circuits (PCs)

PCs encode joint distributions via computational graphs



Probabilistic Circuits (PCs)

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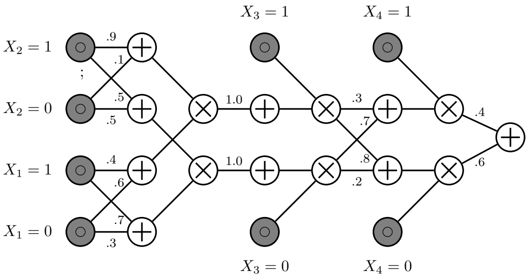

Input nodes are tractable distributions, *e.g., indicator functions* $\mathrm{p}(X_i = 1) = [X_i = 1]$

Probabilistic Circuits (PCs)

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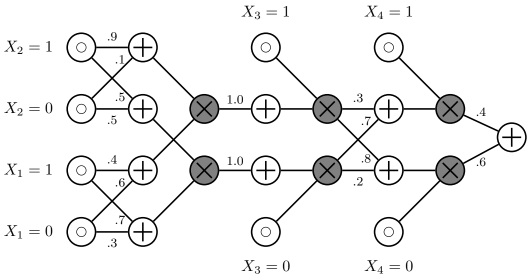
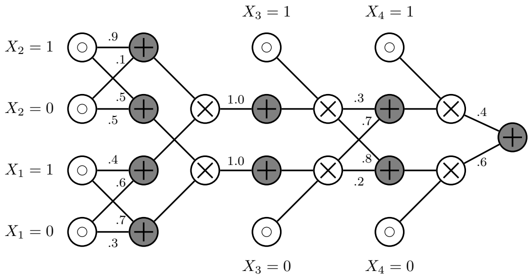


Product nodes are factorizations $\prod_{c \in \mathsf{in}(n)} \mathrm{p}_c(\mathbf{x})$

Probabilistic Circuits (PCs)

PCs encode joint distributions via computational graphs



Sum nodes are mixture models $\sum_{c \in \mathsf{in}(n)} \theta_{n,c}\, \mathrm{p}_c(\mathbf{x})$

- Probabilistic Circuits
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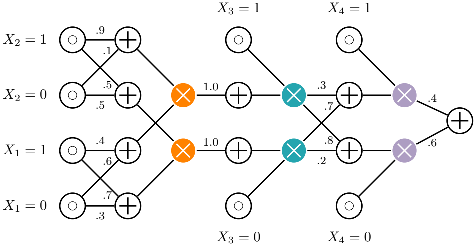
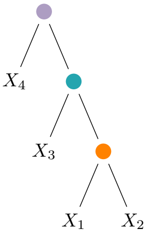
Structured-Decomposable PCs

Every product node over the same set of variables, ***decomposes in the same way...***





...and all allowed ways to decompose are encoded in a ***vtree***

Initialization

How to enforce structured-decomposability?

Start from some ***simple but expressive*** distribution, then ***compile*** it into a PC with a vtree

Q: which distribution?

A: Use a ***Chow-Liu tree*** as the “best” initial PC

- ***easy*** compilation to structured-decomposable PCs
- ***best tree model*** according to KLD *[Chow et al. 1968]*

Structure refinement

How to improve the current structure?

Q: How to build a ***more expressive PC***...while ***preserving properties***?

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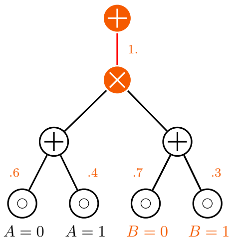
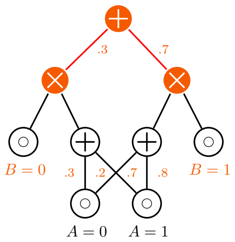




A: by applying the ***split operator***!

Structure refinement

How to split?

How to pick an ***edge*** to split?

$\Longrightarrow$ *take the edge with most samples "flowing" through it*

How to pick a ***variable*** to condition on?

$\Longrightarrow$ *take the variable with the strongest dependencies among others*

Strudel: accurate PCs

DATASET	LEARNPSDD	STRUDEL	SELSPN	SEASPN
NLTCS	-6.03	**-6.06**	-6.03↓	-6.07↑
MSNBC	-6.04	-6.05	-6.04↓	-6.06↑
KDD	-2.17	-2.17	-2.16↓	-2.16↓
PLANTS	-13.49	-13.72	-12.97↓	-13.12↓
AUDIO	-41.51	-42.26	-41.23↓	-40.13↓
JESTER	-54.63	-55.30	-54.38↓	-53.08↓
NETFLIX	-58.53	-58.68	-57.98↓	-56.91↓
ACCIDENTS	-28.291	-29.46	-26.88↓	-30.02↓
RETAIL	-10.92	**-10.90**	-10.88↓	-10.97↑
PUMSB-STAR	-25.40	**-25.28**	-22.66↓	-28.69↓
DNA	-83.02	-87.10	-80.44↓	-81.76↓
KOSAREK	-10.99	**-10.98**	-10.85↓	-11.00↓
MSWEB	-9.93	-10.19	-9.93↓	-10.25↓
BOOK	-36.06	**-35.77**	-36.01↓	-34.91↓
EACHMOVIE	-55.41	-59.47	-55.73↓	-53.28↓
WEBKB	-161.42	**-160.50**	-158.52↓	-157.88↓
ROUTERS-52	-93.30	**-92.38**	-88.48↓	-86.38↓
20NEWS-GRP	-160.43	**-160.77**	-158.68↓	-153.63↓
BBC	-260.24	**-258.96**	-259.35↑	-253.13↓
AD	-20.13	**-16.52**	-16.94↑	-16.77↑

Now we have good single model

Fit the data better and scale more: ***mixture models***

Probabilistic Circuits

Scalable Learning Algorithm

Structured Decomposability and Learning Algorithm

Scale More with Mixtures

Advanced Queries

Conclusions

Strudel: Mixtures of PCs

Difficulties:

- **Structured-decomposable**

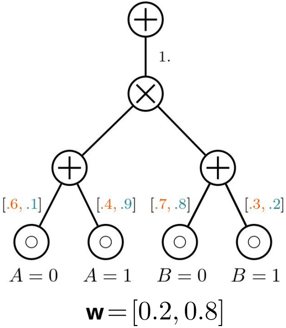
  each component being struct.-decomp....conform to the ***same vtree***

- **Scalable**

  memory, evaluation, learning...

Strudel: Mixtures of PCs

Solution : learn the mixtures ***sharing the same structure***, but having different parameters



- Simple learning and memory efficiency
- Computational efficiency by ***circuit flows***!

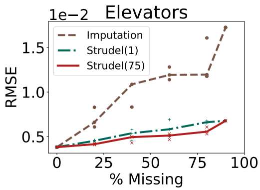
  $\text{logsumexp}(f_{\mathcal{M}}(\mathbf{x})^T \cdot \log(\boldsymbol{\Theta}) + \log(\boldsymbol{w}))$

  E.g., #component 100; #paras each 10,000; N 100,000 : (100,000, 10,000) (10,000, 100) + (1, 100)*

  $\Longrightarrow$ *stop by at poster session :)*

Advanced Queries

Compute the ***expected predictions*** *[Khosravi et al. 2019]*



⇒ ***single*** *probabilistic circuits* ***perform better*** *than the baseline, the* ***mixtures*** *also help further* ***reduce the error***

Conclusions

Probabilistic Circuits can answer advanced queries

$\Rightarrow$ ***structured-decomposable*** *PCs*

We propose a simple and scalable structure learning algorithm

Scale learning:

- Fast heuristic
- Cheap mixtures of circuits sharing the structures
- ***Circuit flows***

References I

- Chow, C and C Liu (1968). “Approximating discrete probability distributions with dependence trees”. In: *IEEE Transactions on Information Theory* 14.3, pp. 462–467.
- Liang, Yitao, Jessa Bekker, and Guy Van den Broeck (2017). “Learning the Structure of Probabilistic Sentential Decision Diagrams”. In: *UAI*.
- Khosravi, Pasha et al. (2019). “Tractable Computation of the Moments of Predictive Models”. In: *Proceedings of NeurIPS*.

Technical Details: Determinism and Circuit Flows

Determinism : for every sum node, at most one of its input is non-zero

Circuit flows : encodes which parameters are activated by different input configurations

Benefits:

- Closed-form MLE
- Fast inference $f_{\mathcal{C}}(\mathbf{x})^T \cdot \log(\boldsymbol{\theta})$

$$[\,0\ \ 1\ \ 0\ \ 0\ \ 1\ \ 0\ \ \ldots] \cdot$$

$$\log([.4\ .6\ .3\ .7\ .8\ .2\ \ldots])$$

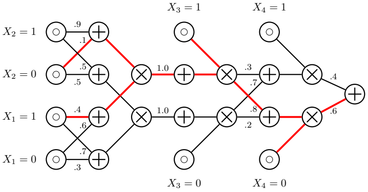


Figure: For input configuration $X_1\!=\!1, X_2\!=\!0, X_3\!=\!1, X_4\!=\!0$, red colors indicate all the **active** edges