AI can learn from data.
But can it learn to reason?

Guy Van den Broeck

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Outline

1. The paradox of learning to reason from data
   
   end-to-end learning

2. Architectures for reasoning
   
   logical + probabilistic reasoning + deep learning
Outline

1. The paradox of learning to reason from data
   \textit{end-to-end learning}

2. Architectures for reasoning
   \textit{logical + probabilistic reasoning + deep learning}
Can Language Models Perform Logical Reasoning?

Language Models achieve high performance on various “reasoning” benchmarks in NLP.

It is unclear whether they solve the tasks following the rules of logical deduction.

**Language Models:**
\[
\text{input} \rightarrow ? \rightarrow \text{Carol is the grandmother of Justin.}
\]

**Logical Reasoning:**
\[
\text{input} \rightarrow \text{Justin in Kristin’s son; Carol is Kristin’s mother; } \rightarrow \text{Carol is Justin’s mother’s mother; if } \text{X is Y’s mother’s mother then X is Y’s grandmother } \rightarrow \text{Carol is the grandmother of Justin.}
\]
Problem Setting: SimpleLogic

The easiest of reasoning problems:

1. **Propositional logic** fragment
   a. bounded vocabulary & number of rules
   b. bounded reasoning depth ($\leq 6$)
   c. finite space ($\approx 10^{360}$)

2. **No language variance**: templated language

3. **Self-contained**
   No prior knowledge

4. **Purely symbolic** predicates
   No shortcuts from word meaning

5. **Tractable** logic (definite clauses)
   Can always be solved efficiently

Facts:
   Alice is *fast*.
   Alice is *normal*.

Rules:
   If Alice is *fast* and *smart*, then Alice is *bad*.
   If Alice is *normal*, then Alice is *smart*.
   If Alice is *normal* and *happy*, then Alice is *sad*.

Query 1: Alice is *bad*. [Answer: True]
Query 2: Alice is *sad*. [Answer: False]

LMs: BERT, T5

Honghua Zhang, Liunian Harold Li, Tao Meng, Kai-Wei Chang and Guy Van den Broeck. *On the Paradox of Learning to Reason from Data*, 2022
Generate textual train and test examples of the form:

Rules: If witty, then diplomatic. If careless and condemned and attractive, then blushing. If dishonest and inquisitive and average, then shy. If average, then stormy. If popular, then blushing. If talented, then hurt. If popular and attractive, then thoughtless. If blushing and shy and stormy, then inquisitive. If adorable, then popular. If cooperative and wrong and stormy, then thoughtless. If popular, then sensible. If cooperative, then wrong. If shy and cooperative, then witty. If polite and shy and thoughtless, then talented. If polite, then condemned. If polite and wrong, then inquisitive. If dishonest and inquisitive, then talented. If blushing and dishonest, then careless. If inquisitive and dishonest, then troubled. If blushing and stormy, then shy. If diplomatic and talented, then careless. If wrong and beautiful, then popular. If ugly and shy and beautiful, then stormy. If shy and inquisitive and attractive, then diplomatic. If witty and beautiful and frightened, then adorable. If diplomatic and cooperative, then sensible. If thoughtless and inquisitive, then diplomatic. If careless and dishonest and troubled, then cooperative. If hurt and witty and troubled, then dishonest. If scared and diplomatic and troubled, then average. If ugly and wrong and careless, then average. If dishonest and scared, then polite. If talented, then dishonest. If condemned, then wrong. If wrong and troubled and blushing, then scared. If attractive and condemned, then frightened. If hurt and condemned and shy, then witty. If cooperative, then attractive. If careless, then polite. If adorable and wrong and careless, then diplomatic. Facts: Alice sensible Alice condemned Alice thoughtless Alice polite Alice scared Alice average
Query: Alice is shy?
Training a transformer on SimpleLogic

(1) Randomly sample facts & rules.
Facts: B, C
Rules: A, B \rightarrow D, B \rightarrow E, B, C \rightarrow F.

(2) Compute the correct labels for all predicates given the facts and rules.

Rule-Priority

Label-Priority

(1) Randomly assign labels to predicates.
True: B, C, E, F.
False: A, D.

Test accuracy for different reasoning depths

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<th>Test</th>
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<th>3</th>
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Has the transformer learned to reason from data?

1. Easiest of reasoning problems (no variance, self-contained, purely symbolic, tractable)
2. RP/LP data covers the whole problem space
3. The learned model has almost 100% test accuracy
4. There exist transformer parameters that compute the ground-truth reasoning function:

   **Theorem 1:** For a BERT model with $n$ layers and 12 attention heads, by construction, there exists a set of parameters such that the model can correctly solve any reasoning problem in SimpleLogic that requires at most $n - 2$ steps of reasoning.

Surely, under these conditions, the transformer has learned the ground-truth reasoning function!
1. If the transformer **has learned** to reason, it should not exhibit such generalization failure.

2. If the transformer **has not learned** to reason, it is baffling how it achieves near-perfect in-distribution test accuracy.

The BERT model trained on one distribution fails to generalize to the other distribution within the same problem space.
Why? Statistical Features

Monotonicity of entailment:
*Any rules can be freely added to the axioms of any proven fact.*

The more rules given, the more likely a predicate will be proven.

\[ \Pr(\text{label} = \text{True} \mid \text{Rule } \# = x) \text{ should increase (roughly) monotonically with } x \]
Model leverages statistical features to make predictions

RP_b downsamples from RP such that Pr(label = True | rule# = x) = 0.5 for all x

<table>
<thead>
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<th>2</th>
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<td>88.3</td>
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</table>

1. Accuracy drop from RP to RP_b indicates that
   the model is using rule# as a statistical feature to make predictions.

2. Potentially countless statistical features

3. Such features are inherent to the reasoning problem, cannot make data “clean”
First Conclusion

Experiments unveil the fundamental difference between

1. learning to reason, and
2. learning to achieve high performance on benchmarks using statistical features.

Be careful deploying AI in applications where this difference matters.

FAQ: *Do bigger transformers solve this problem?* No, already 99% accurate…

FAQ: *Will reasoning emerge?* Perhaps on 99% of human behavior…
Initialize the model with the perfect parameters that simulate the ground-truth reasoning algorithm.

Then SGD will **un-learn the algorithm** that generalizes OOD and again learn statistical shortcuts.

**Theorem 1:** For a BERT model with $n$ layers and 12 attention heads, by construction, there exists a set of parameters such that the model can correctly solve any reasoning problem in SimpleLogic that requires at most $n - 2$ steps of reasoning.
Outline

1. The paradox of learning to reason from data

2. Architectures for reasoning

logical + probabilistic reasoning + deep learning
Generate a sentence using "frisbee", "caught" and "dog", following the given order.
Generate a sentence using "frisbee", "caught" and "dog", following the given order.

After a perfect throw, the frisbee glided through the air, and the dog, with incredible agility, caught it mid-flight.
Generate a sentence using "frisbee", "caught" and "dog", following the given order.

After a perfect throw, the frisbee glided through the air, and the dog, with incredible agility, caught it mid-flight.

That's not correct. Generate a sentence using "frisbee", "caught" and "dog". The keywords should appear in the order as specified.

Here's the correct sentence: The dog caught the frisbee in mid-air, showing off its amazing catching skills.
A frisbee is caught by a dog.

A pair of frisbee players are caught in a dog fight.
What do we have?

Prefix: “The weather is”

Constraint $\alpha$: text contains “winter”

Model only does $p(\text{next-token}|\text{prefix}) =$

<p>| | |</p>
<table>
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<td>cold</td>
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<tr>
<td>warm</td>
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</table>

Train some $q(\cdot|\alpha)$ for a specific task distribution $\alpha \sim p_{\text{task}}$

(amortized inference, encoder, masked model, seq2seq, prompt tuning,...)

Train $q(\text{next-token}|\text{prefix}, \alpha)$
What do we need?

Prefix: “The weather is”

Constraint α: text contains “winter”

Generate from \( p(\text{next-token}|\text{prefix}, \alpha) = \alpha \sum_{\text{text}} p(\text{next-token}, \text{text}, \text{prefix}, \alpha) \)

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<tbody>
<tr>
<td>cold</td>
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<tr>
<td>warm</td>
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Marginalization!
Tractable Probabilistic Models

Tractable Probabilistic Models (TPMs) model joint probability distributions and allow efficient probabilistic inference.

e.g., efficient marginalization:

\[ p_{\text{TPM}}(3\text{rd token} = \text{frisbee}, 5\text{th token} = \text{dog}) \]

Easily understood as tractable probabilistic circuits (see later).

For now… keep it simple… just a Hidden Markov Model (HMM)
Step 1: Distill an HMM $p_{hmm}$ that approximates $p_{gpt}$

1. HMM with 4096 hidden states and 50k emission tokens

2. Data sampled from GPT2-large (domain-adapted), minimizing $KL(p_{gpt} \parallel p_{HMM})$

3. Leverages latent variable distillation for training at scale [ICLR 23]. (Cluster embeddings of examples to estimate latent $Z_i$)

CommonGen: a Challenging Benchmark

Given 3-5 keywords, generate a sentence using all keywords, in any order and any form of inflections. e.g.,

Input: snow drive car

Reference 1: A car drives down a snow covered road.

Reference 2: Two cars drove through the snow.

Constraint $\alpha$ in CNF: \[(w_{1,1} \lor \ldots \lor w_{1,d_1}) \land \ldots \land (w_{m,1} \lor \ldots \lor w_{m,d_m})\]

Each clause represents the inflections for one keyword.
Computing \( p(\alpha \mid x_{1:t+1}) \)

For constraint \( \alpha \) in CNF:

\[
(w_{1,1} \lor \ldots \lor w_{1,d_1}) \land \ldots \land (w_{m,1} \lor \ldots \lor w_{m,d_m})
\]

e.g., \( \alpha = ("swims" \lor "like swimming") \land ("lake" \lor "pool") \)

**Efficient algorithm:**
For \( m \) clauses and sequence length \( n \), time-complexity for HMM generation is \( O(2^m n) \)

**Trick:** dynamic programming with clever preprocessing and local belief updates

GeLaTo
Overview

**Lexical Constraint** $\alpha$: sentence contains keyword "winter"

**Constrained Generation**: $\Pr(x_{t+1} \mid \alpha, x_{1:t} = "the weather is")$

<table>
<thead>
<tr>
<th>$x_{t+1}$</th>
<th>$Pr_{LM}(x_{t+1} \mid x_{1:t})$</th>
<th>$Pr_{TPM}(\alpha \mid x_{t+1}, x_{1:t})$</th>
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<tr>
<td>cold</td>
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<td>warm</td>
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Pre-trained Language Model

Tractable Probabilistic Model

Minimize KL-divergence

GeLaTo
Overview

Lexical Constraint $\alpha$: sentence contains keyword “winter”

Constrained Generation: $\Pr(x_{t+1} | \alpha, x_{1:t} = "the weather is")$

Pre-trained Language Model

| $x_{t+1}$ | $Pr_{LM}(x_{t+1} | x_{1:t})$ |
|-----------|-----------------------------|
| cold      | 0.05                        |
| warm      | 0.10                        |

Tractable Probabilistic Model

| $x_{t+1}$ | $Pr_{TPM}(\alpha | x_{t+1}, x_{1:t})$ |
|-----------|----------------------------------|
| cold      | 0.50                             |
| warm      | 0.01                             |

Minimize KL-divergence

Step 2: Control $p_{gpt}$ via $p_{hmm}$

**Unsupervised**

Language model is not fine-tuned/prompted to satisfy constraints

By Bayes rule:

$$p_{gpt}(x_{t+1} | x_{1:t}, \alpha) \propto p_{gpt}(\alpha | x_{1:t+1}) \cdot p_{gpt}(x_{t+1} | x_{1:t})$$

Assume $p_{hmm}(\alpha | x_{1:t+1}) \approx p_{gpt}(\alpha | x_{1:t+1})$, we generate from:

$$p(x_{t+1} | x_{1:t}, \alpha) \propto p_{hmm}(\alpha | x_{1:t+1}) \cdot p_{gpt}(x_{t+1} | x_{1:t})$$

<table>
<thead>
<tr>
<th>Method</th>
<th>ROUGE-L dev</th>
<th>ROUGE-L test</th>
<th>BLEU-4 dev</th>
<th>BLEU-4 test</th>
<th>CIDEr dev</th>
<th>CIDEr test</th>
<th>SPICE dev</th>
<th>SPICE test</th>
<th>Constraint Satisfaction Coverage</th>
<th>Constraint Satisfaction Success Rate</th>
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<td>GeLaTo</td>
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</table>

Step 2: Control $p_{\text{gpt}}$ via $p_{\text{hmm}}$

**Supervised**

Language model is fine-tuned to perform constrained generation (e.g. seq2seq)

Empirically $p_{\text{HMM}}(\alpha \mid x_{1:t+1}) \approx p_{\text{gpt}}(\alpha \mid x_{1:t+1})$

does not hold well enough;

we view $p_{\text{HMM}}(x_{t+1} \mid x_{1:t}, \alpha)$ and $p_{\text{gpt}}(x_{t+1} \mid x_{1:t})$ as classifiers trained for the same task with different biases; thus we generate from their *weighted geometric mean*:

$$p(x_{t+1} \mid x_{1:t}, \alpha) \propto p_{\text{hmm}}(x_{t+1} \mid x_{1:t}, \alpha)^w \cdot p_{\text{gpt}}(x_{t+1} \mid x_{1:t})^{1-w}$$

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<table>
<thead>
<tr>
<th>Method</th>
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<th>Generation Quality</th>
<th>Constraint Satisfaction</th>
<th>Constraint Satisfaction</th>
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<td><strong>32.9</strong></td>
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</table>
Advantages of GeLaTo:

1. Constraint $\alpha$ is guaranteed to be satisfied:
   for any next-token $x_{t+1}$ that would make $\alpha$ unsatisfiable, $p(x_{t+1} | x_{1:t}, \alpha) = 0$.

2. Training $p_{hmm}$ does not depend on $\alpha$, which is only imposed at inference (generation) time.

3. Can impose additional tractable constraints:
   - keywords follow a particular order
   - keywords appear at a particular position
   - keywords must not appear

Conclusion: you can control an intractable generative model using a tractable probabilistic circuit.
Diffusion models are good at fine-grained details, but not so good at global consistency of generated images.
Tractable control of diffusion models for inpainting
Probabilistic circuits

*computational graphs* that recursively define distributions

\[ p(X_1) = w_1p_1(X_1) + w_2p_2(X_1) \]

\[ p(X_1, X_2) = p(X_1) \cdot p(X_2) \]

\[ \Rightarrow \text{mixtures} \]

\[ \Rightarrow \text{factorizations} \]
From the diffusion model:
Good at generating vivid details

From the probabilistic circuit:
Exact samples – better global coherence

Denoising
\[
p(\tilde{x}_0 | x_t, x_0^k) \propto p_{DM}(\tilde{x}_0 | x_t, x_0^k)\alpha \cdot p_{TPM}(\tilde{x}_0 | x_t, x_0^k)^{1-\alpha}
\]
## High-resolution image benchmarks

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<th>Tasks</th>
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<th>Algorithms</th>
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<td>Tiramisu (ours)</td>
<td>CoPaint</td>
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<td><strong>Average</strong></td>
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Qualitative results on high-resolution image datasets

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<td>Tiramisu (ours)</td>
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Outline

1. The paradox of learning to reason from data
2. Architectures for reasoning
Thanks

This was the work of many wonderful students/postdocs/collaborators!

References: [http://starai.cs.ucla.edu/publications/]