On the Complexity and Approximation of Binary Evidence in Lifted Inference

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- Statistical relational model (e.g., MLN)

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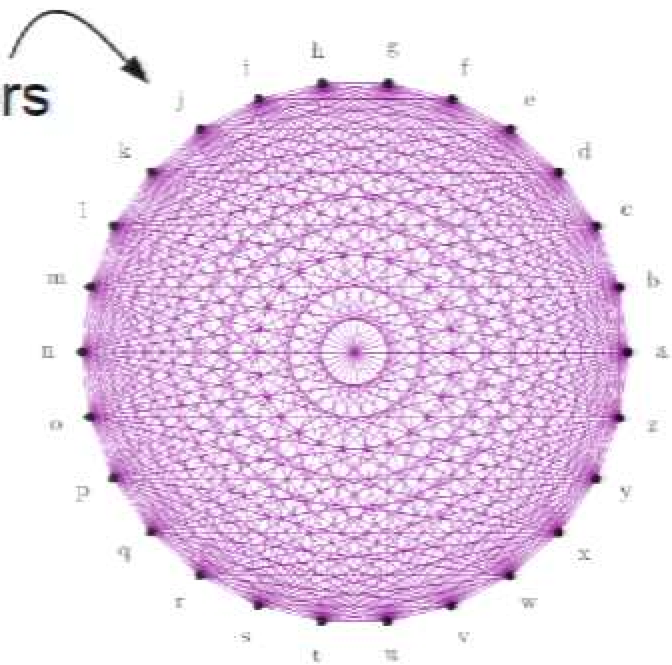
- As a probabilistic graphical model:
  - 26 pages, 728 random variables, 676 factors
  - 1000 pages, 1,002,000 random variables, 1,000,000 factors
- Highly intractable?
- Exploit symmetries:

  Lifted inference in milliseconds!



Complexity of Probabilistic Inference

- What we knew before:
  - No evidence: Efficient (polynomial in domain size/number of pages)
  - Evidence on unary relations: Efficient

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  - Evidence on binary relations: #P-hard

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  - Intuition: Binary evidence breaks symmetries
- New complexity results:
  - Represent binary evidence as Boolean matrix
  - **Boolean Matrix factorization** turns binary into unary
  - Boolean **rank** (size of smallest factorization) is key parameter

Analogy with Treewidth in Bayesian Networks

Bayesian networks: ⟷ SRL Models:

Bayesian networks:

1. Find tree decomposition (e.g., variable elimination order, dtree, jointree)
2. Perform inference

- Exponential in (**tree**)**width** of decomposition
- Polynomial in **size** of Bayesian network

SRL Models:

1. Find Boolean matrix factorization of evidence
2. Perform inference

- Exponential in **Boolean rank** of evidence
- Polynomial in **size** of evidence
- Polynomial in **domain size**

Over-Symmetric Evidence Approximation

- New appoximate lifted inference technique

  Approximate Pr(q|e) by Pr(q|e')

  Pr(q|e') has **more symmetries**, is more liftable

- Instance: **Low-rank** Boolean matrix factorization

$$\mathbf{P} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \approx \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}^T = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

- Lifted MCMC Experiments on WebKB data