On the Paradox of Learning to Reason from Data

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Abstract

Logical reasoning is needed in a wide range of NLP tasks. Can a BERT model be trained end-to-end to solve logical reasoning problems presented in natural language? We attempt to answer this question in a *confined problem space* where there exists a set of parameters that *perfectly simulates* logical reasoning. We make observations that seem to contradict each other: BERT attains near-perfect accuracy on in-distribution test examples while failing to generalize to other data distributions over the exact *same* problem space. Our study provides an explanation for this paradox: instead of learning to emulate the correct reasoning function, BERT has, in fact, learned *statistical features* that inherently exist in logical reasoning problems. We also show that it is infeasible to jointly remove statistical features from data, illustrating the difficulty of learning to reason in general. Our result naturally extends to other neural models (e.g. T5) and unveils the fundamental difference between learning to reason and learning to achieve high performance on NLP benchmarks using statistical features.

1 Introduction

Logical reasoning is needed in a wide range of NLP tasks, including natural language inference (NLI) (Williams et al., 2018; Bowman et al., 2015), question answering (QA) (Rajpurkar et al., 2016; Yang et al., 2018) and common-sense reasoning (Zellers et al., 2018; Talmor et al., 2019). The ability to draw conclusions based on given facts and rules is essential to solving these tasks.[1] Though NLP models, empowered by the Transformer neural architecture (Vaswani et al., 2017), can achieve high performance on task-specific datasets, it is unclear whether they are "reasoning" following the rules of logic. A research question naturally arises: *can neural models be trained end-to-end to conduct logical reasoning in natural language?*

[1] A.k.a., deductive reasoning; in this paper, we do not consider inductive reasoning, where rules need to be learned.

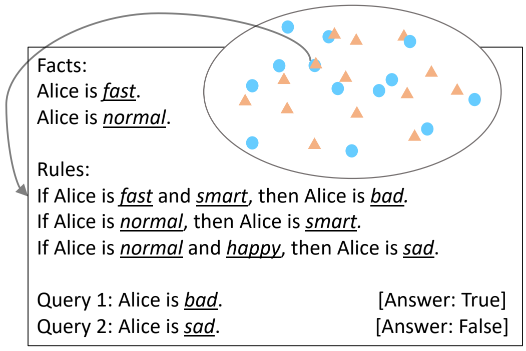


Figure 1: A confined problem space (SimpleLogic) consisting of exponentially many ($\approx 10^{360}$) logical reasoning problems; dots and triangles denote examples sampled from two different distributions over the same problem space.

Following prior work, we attempt to answer this question by training and testing a neural model (e.g. BERT (Devlin et al., 2019)) on a **confined problem space** (see Fig. 1 and Sec. 2) consisting of logical reasoning problems written in English (Johnson et al., 2017; Sinha et al., 2019). Yet, we observe evidences that seemingly lead to a contradiction.

On the one hand, echoing the findings of prior work (Clark et al., 2020; Talmor et al., 2020), we observe evidences that seem to imply that *neural models can learn to reason* (i.e. reliably emulate the correct reasoning function): (E1) examples in the problem space only test model's reasoning ability: they have *no* language variance and require *no* prior knowledge; (E2) we prove by construction that the BERT model has enough capacity to represent the correct reasoning function (Sec 2.2); (E3) the BERT model can be trained to achieve near-perfect test accuracy on data distributions covering the whole problem space.

On the other hand, we observe a contradictory phenomenon: the models attaining near-perfect accuracy on one data distribution do not generalize to other distributions within *the same problem space* (Sec. 3). Since the correct reasoning function does not change across data distributions, it follows that *the model has not learned to reason*.

The paradox lies in that if a neural model *has* learned reasoning, it should not exhibit such a generalization failure; if the model *has not* learned reasoning, it is baffling how it manages to achieve near-perfect test accuracy on training distributions that cover the entire problem space. Note that what we observed is not a common case of out-of-distribution (OOD) generalization failure: (1) our problem space is confined and simple (see E1, E2); (2) the correct reasoning function is invariant across data distributions; on the contrary, discussions about OOD generalization often involve open problem spaces (Lin et al., 2019; Gontier et al., 2020; Yin et al., 2020; Wald et al., 2021) and domain/concept mismatch between training and testing distribution (Yin et al., 2021; Koh et al., 2021).

Upon further investigation, we provide an explanation for this paradox: the model attaining high accuracy **only** on in-distribution test examples **has not learned to reason**. In fact, the model has learned to use *statistical features* in logical reasoning problems to make predictions rather than to emulate the correct reasoning function.

Our first observation is that even the simplest statistic of a reasoning problem can give away significant information about the true label (Sec. 4.1): for example, by only looking at the number of rules in a reasoning problem, we can predict the correct label better than a random guess. Unlike dataset biases/artifacts identified in typical NLP datasets, which are often due to biases in the dataset collection/annotation process (Gururangan et al., 2018; Clark et al., 2019; He et al., 2019), statistical features **inherently** exist in reasoning problems and are not specific to certain data distributions. We show that statistical features can hinder model generalization performance (Sec. 4.2); moreover, we argue that there are potentially countless statistical features and demonstrate that it is computationally expensive to jointly remove them from training data (Sec. 4.3).

Our study implies the difficulty of learning to reason from data: while a model always tends to learn statistical features, it is difficult to construct a logical reasoning dataset that exhibits no statistical features. Though we use BERT as the running example throughout this paper, our argument assumes little about model architecture and naturally extends to other neural models. This intuition is supported by experiments with T5 (Raffel et al., 2020), which exhibits behaviors similar to BERT.

Our findings unveil the fundamental difference between "learning to reason" and "learning to attain high performance on NLP benchmarks." Learning statistical features is not always undesirable; in fact, for most NLP tasks, one of the major goal for a neural model is to learn statistical patterns: for example, in sentiment analysis (Maas et al., 2011), a model is *expected* to learn the strong correlation between the occurrence of the word "happy" and the positive sentiment. However, for logical reasoning, even though countless statistical features inherently exist, models should not use them to make predictions. Caution should be taken when we seek to train neural models end-to-end to solve logical reasoning tasks in NLP that involve prior knowledge and are presented with language variance (Welleck et al., 2021; Yu et al., 2020), which could potentially lead to even stronger statistical features, as demonstrated by Elazar et al. (2021) and McCoy et al. (2019).

Code/data will be publicized upon acceptance.

2 SimpleLogic: A Simple Problem Space for Logical Reasoning

We define *SimpleLogic*, a class of logical reasoning problems based on propositional logic, as a controlled testbed for testing neural models' ability to conduct logical reasoning. SimpleLogic only contains deductive reasoning examples. To simplify the problem, we remove language variance by representing the reasoning problems in a templated language and limit their complexity (e.g., examples have limited input lengths and reasoning depths).

Solving SimpleLogic does not require significant model capacity. We show that the BERT model (Devlin et al., 2019) has more than enough model capacity to solve SimpleLogic by constructing a parameterization of BERT that can solve all instances in SimpleLogic (Sec. 2.2).

2.1 Problem Space Definition

Before defining SimpleLogic, we introduce some basics for propositional logic. In general, reasoning in propositional logic is NP-complete (Cook, 1971); hence, we only consider propositional reasoning with *definite clauses*. A definite clause is a *rule* of the form $A_1 \wedge A_2 \wedge \cdots \wedge A_n \rightarrow B$, where A_is and B are *predicates* that take values in "True" or "False"; we refer to the left hand side of a rule as its *body* and the right hand side as its *head*. In particular, a definite clause is called a *fact* if its body is empty (i.e. $n = 0$). A *propositional theory* T is a set of rules and facts, and we say a predicate Q *can be proved* from T if either (1) Q is given in T as a fact or (2) $A_1 \wedge \cdots \wedge A_n \rightarrow Q$ is given in T as a rule where A_is can be proved.

Each example in SimpleLogic is a propositional reasoning problem that only involves definite clauses. In particular, each example is a tuple (*facts*, *rules*, *query*, *label*) where (1) *facts* is a list of predicates that are known to be True, (2) *rules* is a list of rules represented as definite clauses, (3) *query* is a single predicate, and (4) *label* is either True or False, denoting whether the query predicate can be proved from *facts* and *rules*. Figure 1 shows such an example. Furthermore, we enforce simple constraints to control the difficulty of the problems. For each example in SimpleLogic, we require that:

- the number of predicates (#pred) that appear in facts, rules and query ranges from 5 to 30, and all predicates are sampled from a fixed vocabulary containing 150 adjectives such as "happy" and "complicated"; note that the predicates in SimpleLogic have **no** semantics;
- $0 \leq$ the number of rules (#rule) $\leq 4\times$ #pred; the body of each rule contains 1 to 3 predicates; i.e. $A_1 \wedge \ldots \wedge A_n \rightarrow B$ with $n > 3$ is not allowed;
- $1 \leq$ the number of facts (#fact) $\leq$ #pred;
- the reasoning depth[2] required to solve an example ranges from 0 to 6.

We use a simple template to encode examples in SimpleLogic as English input. For example, we use "*Alice is X.*" to represent the fact that X is True; we use "*A and B, C.*" to represent the rule $A \wedge B \rightarrow C$; we use "*Query: Alice is Q.*" to represent the query predicate Q. We concatenate *facts*, *rules* and *query* as "[CLS] *facts*. *rules* [SEP] *query* [SEP]" and supplement it to BERT to predict the correct label.

2.2 BERT Has Enough Capacity to Solve SimpleLogic

In the following, we show that BERT has enough capacity to solve all the examples in SimpleLogic. In particular, we explicitly construct a parameterization for BERT such that the fixed-parameter model solves all problem instances in SimlpleLogic. Note that we only prove the existence of such a parameterization, but do not discuss whether it can be learned from data until Sec. 3.

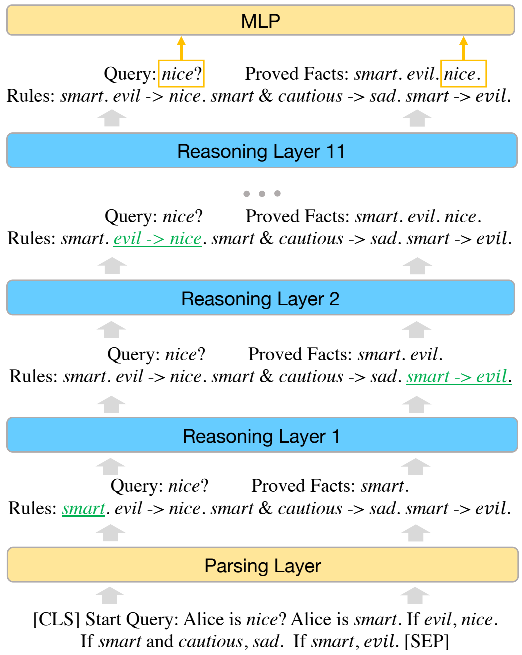


Figure 2: A BERT-base model that simulates the forward-chaining algorithm. The first layer parses text input into the desired format. Each reasoning layer performs one step of forward-chaining, adding some predicates to the Proved Facts, and the rules being used are underlined in green; e.g. Reasoning Layer 1 use the rule "smart $\rightarrow$ evil" to prove the predicate *evil*.

Theorem 1. *For BERT with n layers, there exists a set of parameters such that the model can correctly solve any reasoning problem in SimpleLogic that requires $\leq n - 2$ steps of reasoning.*

Proof Sketch. To prove this theorem, we construct a fixed set of parameters for BERT to simulate the forward-chaining algorithm. As illustrated in Figure 2, our construction solves a logical reasoning example in a layer-by-layer fashion. The 1st layer of BERT parses the input sequence into the desired format. Layer 2 to layer 10 are responsible for simulating the forward chaining algorithm: each layer performs one step of reasoning, updating the True/False label for predicates. The last layer also performs one step of reasoning, while implicitly checking if the query predicate is proved and feeding the result to an MLP. The parameters are the same across all layers except for the Parsing Layer. We refer readers to the Appendix for details. □

We implemented the construction in PyTorch,

[2] For a query with label *True*, its reasoning depth is given by the depth of the shallowest proof tree; for a query with label *False*, its reasoning depth is the maximum depth of the shallowest failing branch in all *possible* proof trees.

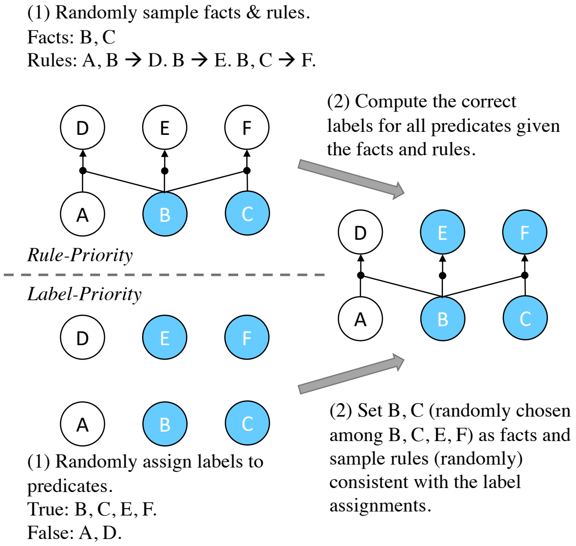


Figure 3: An illustration of a logical reasoning problem (right) in SimpleLogic being sampled by Rule-Priority (RP) and Label-Priority (LP), respectively. Predicates with label *True* are denoted by filled circles.

following the exact architecture of the BERT-base model. As supported by the theorem, the "constructed BERT" solves all the problems in SimpleLogic of reasoning depth ≤ 10 with 100% accuracy.

3 BERT Fails to Learn to Solve SimpleLogic

Next, we study whether it is possible to train a neural model (e.g., BERT) to reason on SimpleLogic. We follow Clark et al. (2020) to randomly sample examples from the problem space and train the BERT model on a large amount of sampled data.

3.1 Sampling Examples from SimpleLogic

When sampling examples from a finite domain, one naive approach is to uniformly sample from the domain. However, uniform sampling is *not desirable*: as described in Sec. 2.1, examples in SimpleLogic have #pred ranging from 5 to 30 and #rule ranging from 0 to 4 $\times$ #pred, as the number of combinations with #pred = 30 and #rule = 120 is significantly larger than other settings, if follows that over 99.99% of the examples generated by uniform sampling would have 30 predicates and 120 rules. This is a serious problem as we expect our training set to contain examples of different #pred, #fact and #rule. Hence, we instead consider the following two intuitive ways of sampling examples:

Rule-Priority (RP). In Rule-Priority, we first randomly sample #pred, #fact and #rule uniformly at random from $[5, 30]$, $[1, \#\text{pred}]$ and $[1, 4 \times \#\text{pred}]$ respectively, ensuring that all three aspects are covered by a non-trivial number of examples. Then, we randomly sample some predicates, facts and rules based on the given #pred, #rule and #fact. The query is also randomly sampled, and its label is computed by forward-chaining based on the given facts and rules.

Lable-Priority (LP). In Rule-Priority, we first randomly generate rules and facts, which then determines the label for each predicate. In Label-Priority (LP), we consider generating examples in the "reversed" order: we first randomly assign a True/False label to each predicate and then randomly sample some rules and facts that are *consistent* with the pre-assigned labels.

Figure 3 shows an example that illustrates the two sampling methods. Both LP and RP are general, and they cover the whole problem space. We refer readers to the Appendix for further details about the sampling algorithms.

3.2 BERT Trained on Randomly Sampled Data Cannot Generalize

Following the two sampling regimes described above, we randomly sample two sets of examples from SimpleLogic: for each reasoning depth from 0 to 6, we sample $40k$ examples from SimpleLogic via algorithm RP and aggregate them as dataset RP, which contains $280k$ examples in total; we then split it as training/validation/test set. We use the same procedure to generate dataset RP. We train a BERT-base model (Devlin et al., 2019) on RP and LP, respectively. See details in the appendix.

BERT performs well on the training distributions. The first and last rows of Table 5 show the test accuracy when the test and train examples are sampled by the same algorithm (e.g., for row 1, the model is trained in the RP training set and tested in the RP test set): the models achieve near-perfect performance similar to the findings in prior work (Clark et al., 2020). Both sampling algorithms are general in the sense that every instance in SimpleLogic has a positive probability to be sampled; hence, the intuition is that the model has learned to emulate the correct reasoning function.

BERT fails to generalize. However, at the same time, we observe a rather counterintuitive finding: the test accuracy drops significantly when the train and test examples are sampled via different algo-

Train	Test	0	1	2	3	4	5	6
RP	RP	99.9	99.8	99.7	99.3	98.3	97.5	95.5
	LP	99.8	99.8	99.3	96.0	90.4	75.0	57.3
LP	RP	97.3	66.9	53.0	54.2	59.5	65.6	69.2
	LP	100.0	100.0	99.9	99.9	99.7	99.7	99.0

Table 1: Test accuracy on LP/RP for the BERT model trained on LP/RP; the accuracy is shown for test examples with reasoning depth from 0 to 6. BERT trained on RP achieves almost perfect accuracy on its test set; however the accuracy drops significantly when it's tested on LP (vice versa).

Test	0	1	2	3	4	5	6
RP&LP	99.9	99.9	99.8	99.4	98.8	98.1	95.6
LP*	98.1	97.2	92.5	80.3	65.8	55.6	55.2

Table 2: BERT trained on a mixture over RP and LP fails on LP*, a test set that slightly differs from LP.

rithms. Specifically, as shown in the second and third rows of Table 5, the BERT model trained on RP fails drastically on LP, and vice versa. Since the correct reasoning function does not change across different data distributions, this generalization failure indicates BERT is has not learned to conduct logical reasoning. A subsequent question naturally arises: *can the model learn to reason if we train the model on both RP and LP?*

Training on both RP and LP is not enough. We train BERT on the mixture of RP and LP, and BERT again achieves nearly perfect test accuracy. Can we now conclude that BERT has learned to approximate the correct reasoning function? We slightly tweak the sampling algorithm of LP by increasing the expected number of alternative proof trees to generate LP*. Unfortunately, we observe that the model performance again drops significantly on LP* (Table 2); such a result resembles what we observed in Table 5. In fact, we find *no evidence* that consistently enriching the training distribution will bring a transformative change such that the model can learn to reason.

Discussion. The experiments above reveal a pattern of generalization failure: *if we train the model on one data distribution, it fails almost inevitably on a different distribution.* In other words, the model seems to be emulating an incorrect "*reasoning function*" specific to its training distribution.

4 BERT Learns Statistical Features

To this point, we have shown that a BERT model achieving high in-distribution accuracy does not learn the correct reasoning function. In this section, we seek to provide an explanation for this peculiar generalization failure. Our analysis suggests that even the simplest statistics of reasoning problems can provide significant information about their labels, which we denote as *statistical features*. Such statistical features are **inherent** to the task of logical reasoning rather than a problem with specific datasets. When BERT is trained on data with statistical features, it tends to make predictions based on such features rather than learning to emulate the correct reasoning function; thus, BERT fails to generalize to the whole problem space. However, unlike the shallow shortcuts found in other typical NLP tasks, such statistical features can be countless and extremely complicated, and thus very difficult to be removed from training data.

4.1 Statistical Features Inherently Exists

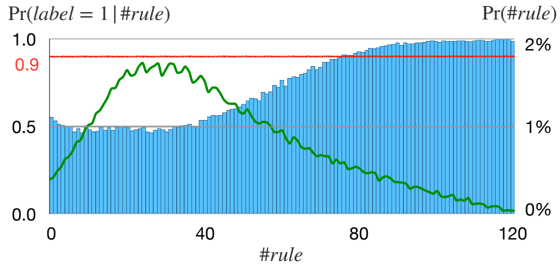
What is a statistical feature? If a certain statistic of an example has a strong correlation with its label, we call it a *statistical feature*.

As an illustrating example, we consider the number of rules in a reasoning problem (#rule). As shown in Figure 4a, the #rule for reasoning problems in RP exhibit a strong correlation with their labels: when #rule > 40, the number of positive examples exceeds 50% by large margins; formally, $\Pr_{e\sim\text{RP}}(\text{label(e)} = 1 \mid \text{\#rule}(e) = x) > 0.5$ for $x > 40$, which makes it possible for the model to guess the label of an example with relatively high accuracy by only using its #rule. Hence, we call #rule a statistical feature for the dataset RP.

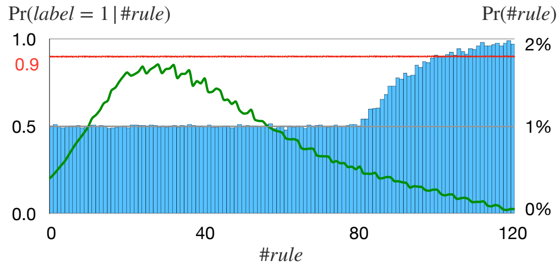
Statistical features are inherent to logical reasoning problems. Continuing with our example, we show that #rule *inherently* exists as a statistical feature for logical reasoning problems in general; that is, it is not specific to the RP dataset. Consider the following property about logical entailment:

Property (Monotonicity of entailment)**.** *Any facts and rules can be freely added to the hypothesis of any proven fact.*

It follows that, intuitively, given a fixed set of predicates and facts, any predicate is more likely to be proved when more rules are given, that is, $\Pr(\text{label(e)} = 1 \mid \text{\#rule}(e) = x)$ should increase roughly monotonically as x increases. Since this



(a) RP: $\Pr(\text{label} = 1 \mid \#\text{rule}) > 0.5$ for #rule > 40.



(b) RP_balance: $\Pr(\text{label} = 1 \mid \#\text{rule}) \approx 0.5$ for #rule ≤ 80.

Figure 4: $\Pr(\text{label} = 1 \mid \#\text{rule})$ (the blue columns) and $\Pr(\#\text{rule})$ (the green curves) for RP and RP_balance, respectively. After removing #rule as a statistical feature (RP_balance), $\Pr(\text{label} = 1 \mid \#\text{rule})$ approaches 0.5 for $\#rule \leq 80$ while $\Pr(\#\text{rule})$ does not change.

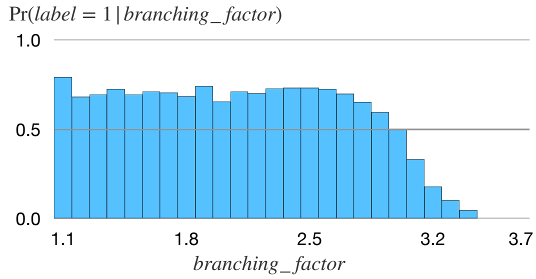


Figure 5: For RP, $\Pr(\text{label} = 1 \mid \text{branching\_factor})$ decreases as branching_factor increases.

intuition assumes nothing about data distributions, it follows that such statistical patterns should naturally exist in any dataset that is not adversarially constructed. In addition to RP, we also verify that both LP and the uniform distribution exhibit similar statistical patterns, which we refer readers to Appendix for further details.

Statistical features are countless. In addition to #rule, numerous statistical features potentially exist. For example, as facts can be seen as special form of rules, it follows from previous argument that #fact is also positively correlated with labels. Statistical features can be more complicated than just #rule or #fact. For example, the average number of predicates in rules of a reasoning problem can also leak information about its label. Note that the right-hand side of a rule is only proved if all predicates on its left-hand side are proved. Then, it is immediate that rules of the form $A, B, C \rightarrow D$ are less likely to be "activated" than rules of the form $A \rightarrow D$. Following this intuition, we can define the following statistic: for an example e, let

$$\text{branching\_factor}(e) := \frac{\#\text{fact}(e) + \sum_{\text{rule} \in e} \text{length of rule}}{\#\text{fact}(e) + \#\text{rule}(e)}.$$

In this definition, we compute the average number of predicates in the rules, where facts are treated as rules with one predicate.[3] Our intuition suggests that the larger the branching_factor, the less likely an example will be positive; we verify that this intuition holds for RP, as shown in Figure 5. Just like #rule, we observe that branching_factor is also a statistical feature for LP and the uniform distribution; see details in Appendix.

Now we have shown that though there are simple statistical features like #rule, some (e.g. branching_factor) can be less intuitive to call to mind; in light of this, it is not hard to imagine that some statistical features can be so complex that they cannot even be manually constructed by humans. In particular, statistical features can also be *compositional*: one can define a *joint* statistical feature by combining multiple ones (e.g., branching_factor and #rule), which further adds to the complexity. Thus, it is infeasible to identify all statistical features.

4.2 Statistical Features Inhibit Model Generalization

Having verified that statistical features inherently exist for logical reasoning problems, in this section we study how they affect the model behavior. We show that (1) when statistical features are presented in training distributions, BERT tends to utilize them to make predictions; (2) after removing **one** statistical feature from training data, the model generalizes better. It follows that statistical features can hinder the model from learning the correct reasoning function, explaining the generalization failure we observed in Section 3.

Example: removing one statistical feature. We use #rule as an example to illustrate how to remove statistical features from a training dataset $\mathcal{D}$;

[3]Branching_factor: with more predicates on the left-hand side of the rules, the proof tree has more branches.

in particular, there are three criteria that we need to satisfy: (1) label is balanced for the feature; (2) the marginal distribution of the feature remains unchanged; (3) the dataset size remains unchanged.

Formally, our first goal is to sample $\mathcal{D}' \subset \mathcal{D}$ such that, for all x:

$$\Pr_{e\sim\mathcal{D}'}(\text{label}(e) = 1 \mid \#\text{rule}(e) = x) = 0.5$$

Intuitively, this equation says that on $\mathcal{D}'$, one cannot do better than 50% by only looking at the #rule of an example. Specifically, for all possible values of x, if $\Pr_{e\sim\mathcal{D}}(\text{label}(e)=1 \mid \#\text{rule}(e)=x) > 0.5$, we drop some positive examples with #rule = x from $\mathcal{D}$; otherwise, we drop some negative examples.

However, we would not meet the second criterion by naively dropping the minimum number of examples; consider the following statistics for RP:

#rule	before drop #examples / positive %	after drop #examples / positive %
38	6860 / 49.9%	6822 / 50.0%
80	2322 / 92.7%	339 / 50.0%

As shown in the table, if we naively drop the minimum number of examples from RP such that Equation 1 is satisfied, we will be left with only 339 examples with #rule = 80, where the number (6822) of examples with #rule = 38 remains unchanged. This could be a serious issue in terms of dataset *coverage*: examples with some particular #rule will dominate $\mathcal{D}'$ and there will not be enough examples for other #rule. Recall that this is also the reason we choose RP/LP over uniform sampling to generate our datasets (Sec. 3.1). Hence, we also need to make sure that as we remove statistical features from $\mathcal{D}$, their marginal distributions in $\mathcal{D}'$ stay close to $\mathcal{D}$:

$$\Pr_{e\sim\mathcal{D}'}(\#\text{rule}(e)) = \Pr_{e\sim\mathcal{D}}(\#\text{rule}(e)).$$

In this way, $\mathcal{D}'$'s coverage of examples with different #rule remains the same as $\mathcal{D}$.

When both criteria (1) and (2) are satisfied, the size of $\mathcal{D}'$ will be *much smaller* than $\mathcal{D}$ and the ratio $k = |\mathcal{D}|/|\mathcal{D}'|$ can be estimated from $\min_x \Pr_{e\sim\mathcal{D}}(\text{label}(e)=1 \mid \#\text{rule}(e)=x)$. Hence, to make sure that criterion (3) is met, that is the size of $\mathcal{D}'$ is the same as $\mathcal{D}$, we need to pre-sample $k \times \mathcal{D}$ and obtain $\mathcal{D}'$ by down-sampling.

Following this approach, by down-sampling from $k \times$ RP, we construct RP_balance, where #rule is no longer a statistical feature. A rough estimation shows that if we were to balance $\Pr_{e\sim\text{RP}}(\text{label}(e) = 1 | \#\text{rule}(e) = x)$ for x up to 110, the ratio $k > 100$, that is, we need to spend over 100x running time (200 hours on a 40-core CPU) to pre-sample roughly 56 million examples; the computational cost would be even more expensive if we want to completely remove #rule as a statistical feature. Hence, we only balance this conditional probability for $0 \le x \le 80$, which takes 10x running time (20 hours on a 40-core CPU) to pre-sample 5.6 million examples. Not balancing the label for $x > 80$ is acceptable as 90% of the examples in RP have #rule ≤ 80. We train the BERT model on RP_balance, and the results are reported in Table 6.

Train	Test	0	1	2	3	4	5	6
RP_b	RP	99.8	99.7	99.7	99.4	98.5	98.1	97.0
	RP_b	99.4	99.6	99.2	98.7	97.8	96.1	94.4
	LP	99.6	99.6	99.6	97.6	93.1	81.3	68.1
RP	RP	99.9	99.8	99.7	99.3	98.3	97.5	95.5
	RP_b	99.0	99.3	98.5	97.5	96.7	93.5	88.3
	LP	99.8	99.8	99.3	96.0	90.4	75.0	57.3

Table 3: The model trained on RP performs worse on RP_balance (RP_b). This indicates that the model is using #rule as a statistical feature to make predictions.

BERT uses statistical features to make predictions. As shown in Table 6, BERT trained on RP shows large performance drop when tested on RP_balance, while BERT trained on RP_balance shows even better performance on RP than RP-trained BERT. Since RP_balance is down-sampled from RP, the accuracy drop from RP to RP_balance can only be explained by that BERT trained on RP is using #rule to make predictions.

Removing statistical features helps generalization. As shown in Table 6, compared to RP-trained BERT, BERT trained on RP_balance achieves higher accuracy when tested on LP; in particular, for examples with reasoning depth 6, the model trained on RP_balance attains an accuracy of 68.1%, approximately 10% higher than the model trained on RP. This is a clear signal that when #rule is removed as a statistical feature, the model generalizes better, suggesting that statistical features can hinder the generalization of the model.

Statistical features explain the paradox. Now we have a good explanation for the paradox: on the first hand, as we have discussed in Section 4.1, statistical features can be arbitrarily complex and powerful neural models can identify and use them to achieve high in-distribution accuracy; on the

X	Pr(label = 1 | X)	k×
f = 15	0.908	5.5
f = 15, b ∈ [2.65,2.75]	0.975	20.0
f = 15, b ∈ [2.65,2.75], r = 58	0.991	55.6

Table 4: Jointly removing statistical features is difficult; e.g. second row shows: we need to sample *at least* 20 × RP to balance Pr(label = 1 | f = 15, b ∈ [2.65, 2.75]).

other hand, since the correlations between statistical features and labels can change as the data distribution changes, the model that relies on statistical features to make predictions does not generalize to out-of-distribution examples.

More importantly, as our argument assumes little about model architectures/pre-training procedures, most of our conclusions should also hold for other neural models. This hypothesis is supported by experiments with T5 (Raffel et al., 2020), which exhibits behaviors similar to BERT: (1) the T5 model attaining near-perfect accuracy on the training distribution fails catastrophically on the other distributions; (2) the T5 model generalizes better after #rule is removed from RP, suggesting that it is using #rule to make predictions. See Appendix for more details.

4.3 On the Dilemma of Removing Statistical Features

We show that though removing one statistical feature (e.g., #rule) from training data can benefit model generalization, it is computationally infeasible to jointly remove multiple statistical features.

In the previous section, when we were trying to remove the #rule from RP, we could only afford to remove it for 90% of the examples. The general idea is that if a statistical feature X has a very strong correlation with the label on some dataset $\mathcal{D}$, i.e. $\Pr_{e\sim\mathcal{D}}(\text{label}(e) = 1 \mid X(e) = x)$ is very close to 1 or 0, then we would need to sample a lot of examples to have a balanced set.

The combination of multiple statistical features can give stronger signal about the label than the individual ones; thus it is even harder to jointly remove them. For example, we consider removing three statistical features from RP: #fact (f), branching_factor (b) and #rule (r). As shown in Table 4, as we try to jointly remove more statistical features X, $\Pr(\text{label} = 1|X)$ becomes more unbalanced; in particular, as we progressively remove #fact, branching_factor and #rule, the minimum times of examples we need to sample grows roughly exponentially: $5.5 \to 20.0 \to 55.6$. Note that we are only considering balancing *one* setting (#fact = 15, braching_factor ∈ [2.65,2.75], #rule = 58); for some other settings, the conditional probability can be more unbalanced, requiring us to pre-sample even more examples.

5 Related Work

Prior work contextualizes the problem of logical reasoning by proposing reasoning-dependent datasets and studies solving the tasks with neural models (Johnson et al., 2017; Sinha et al., 2019; Yu et al., 2020; Liu et al., 2020; Tian et al., 2021). However, most studies focus on solving a single task, and the datasets are either designed for a specific domain (Johnson et al., 2017; Sinha et al., 2019), or have confounding factors such as language variance (Yu et al., 2020); they cannot be used to strictly or comprehensively study the logical reasoning abilities of models.

Another line of research focuses on leveraging deep neural models to solve logical problems. For example, SAT solving (Selsam et al., 2019), maxSAT (Wang et al., 2019), temporal logical problems (Hahn et al., 2021), DNF counting (Crouse et al., 2019), learning embeddings for logical formula (Crouse et al., 2019; Abdelaziz et al., 2020) and mathematical problems (Saxton et al., 2019; Lample and Charton, 2020). In this work, we focus only on deductive reasoning, which is a general and fundamental class of reasoning problems. Xu et al. (2019) develops a theoretical framework to characterize how well neural models can generalize on different reasoning tasks. Beyond deep learning, Darwiche and Marquis (2002) and Khardon and Roth (1997) studies the tractability of reasoning and learning to reason with propositional logic.

6 Conclusion

In this work, we study whether language models can learn to conduct logical reasoning by end-to-end training. We report and provide explanation to a seemingly contradictory phenomenon: while models can attain near-perfect test accuracy on training distributions, they fail catastrophically on other distributions; we demonstrate that they have learned to exploit statistical features rather than to emulate the correct reasoning function. Our study suggests that training on datasets might not be sufficient for model to learn certain complex behaviors such as reasoning and planning.

Limitations

In Sec. 4.3 we discuss the difficulty of jointly removing statistical features from data via sampling, yet we do not study how to develop more efficient algorithms to generate dataset that has no statistical features. We do not perform the analysis on real-world NLP reasoning benchmarks as they often involve factors irrelevant to reasoning itself. Yet real-world logical reasoning benchmarks (Patel et al., 2021; Welleck et al., 2021; Yu et al., 2020) are strictly more difficult than SimpleLogic as models also need to handle language variance and incorporate prior knowledge. If neural models cannot learn to solve SimpleLogic, it is questionable whether they can learn to solve the strictly more complicated counterpart in the real-world.

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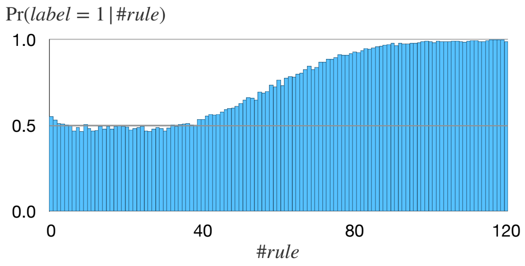
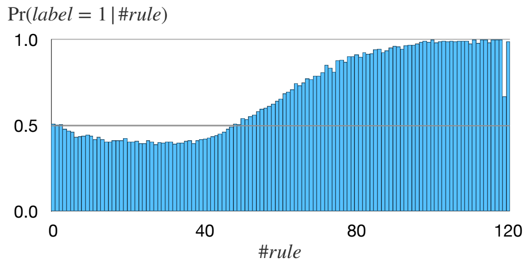
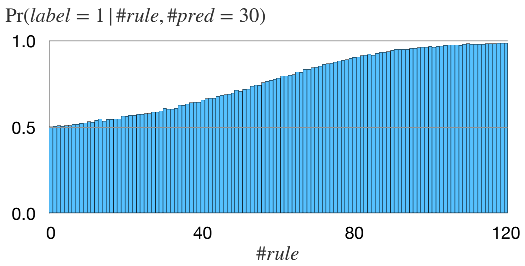
A Statistical Features in Different Data Distributions



(a) Statistics for examples generated by Rule-Priority (RP).

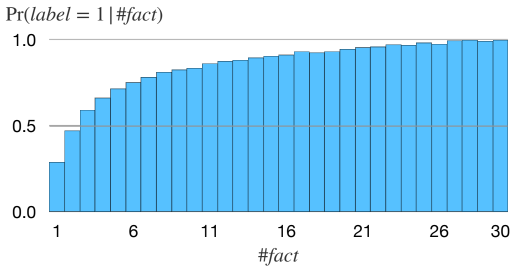


(b) Statistics for examples generated by Label-Priority (LP).

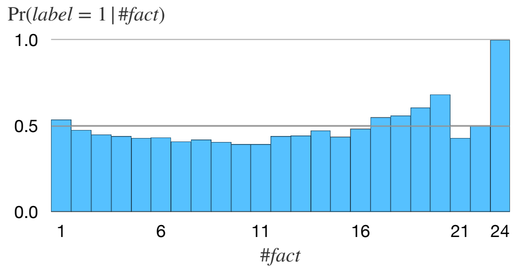


(c) Statistics for examples generated by uniform sampling; we only consider examples with #pred = 30 as a good-enough approximation: over 99% of the examples generated by uniform sampling have #pred = 30.

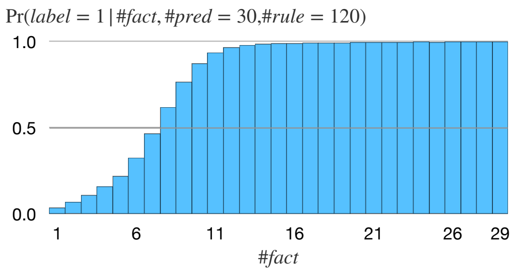
Figure 6: #rule is a statistical feature for RP, LP and the uniform distribution. Even though $\Pr(\text{label} = 1 | \text{#rule})$ increases as #rule increases for all three distributions, it follows a slightly different pattern for each distribution; that is to say, the correlation between #rule and the label changes as the underlying data distribution changes, which explains the generalization failure we observed.



(a) Statistics for examples generated by Rule-Priority (RP).

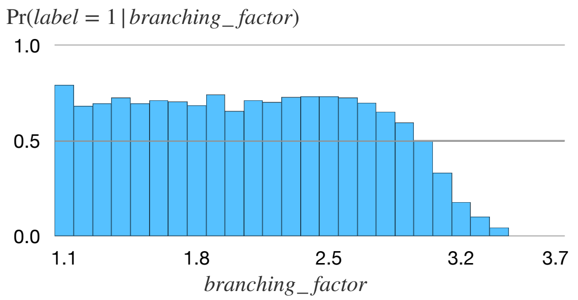


(b) Statistics for examples generated by Label-Priority (LP).

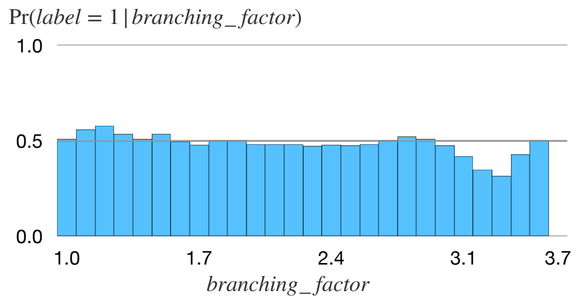


(c) Statistics for examples generated by uniform sampling; we only consider examples with #pred = 30 and #rule = 120 as a good-enough approximation: over 99% of the examples generated by uniform sampling have #pred = 30 and #rule = 120.

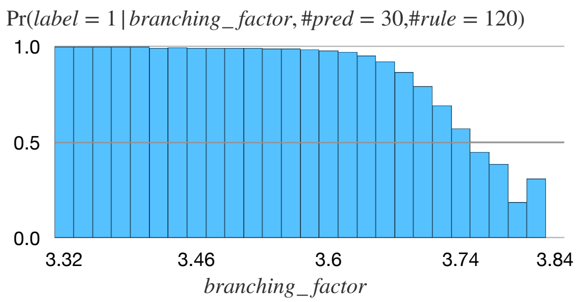
Figure 7: #fact is a statistical feature for RP, LP and the uniform distribution.



(a) Statistics for examples generated by Rule-Priority (RP).



(b) Statistics for examples generated by Label-Priority (LP).



(c) Statistics for examples generated by uniform sampling; we only consider examples with #pred = 30 and #rule = 120 as a good-enough approximation: over 99% of the examples generated by uniform sampling have #pred = 30 and #rule = 120.

Figure 8: branching_factor is a statistical feature for RP, LP and the uniform distribution.

B Experiments with T5

Train	Test	0	1	2	3	4	5	6
RP	RP	100.0	100.0	99.9	99.8	99.3	99.3	98.8
	LP	100.0	99.9	99.9	99.3	96.0	87.7	75.7
LP	RP	100.0	99.6	83.5	56.6	55.5	59.4	65.7
	LP	100.0	100.0	100.0	99.9	99.9	99.7	99.5

Table 5: Test accuracy on LP/RP for the T5 model trained on LP/RP; the accuracy is shown for test examples with reasoning depth from 0 to 6. T5 trained on RP achieves almost perfect accuracy on its test set; however the accuracy drops significantly when it's tested on LP (vice versa), suggesting that T5 has not learned the correct reasoning function.

Train	Test	0	1	2	3	4	5	6
RP_b	RP	99.8	99.7	99.7	99.4	98.5	98.1	97.0
	RP_b	100.0	100.0	99.9	99.8	99.6	99.3	98.9
	LP	100.0	99.9	99.9	99.1	97.0	89.3	80.8
RP	RP	100.0	100.0	99.9	99.8	99.3	99.3	98.8
	RP_b	100.0	99.9	99.8	99.5	99.0	98.5	97.6
	LP	100.0	99.9	99.9	99.3	96.0	87.7	75.7

Table 6: The T5 model trained on RP_balance exhibits better generalization performance on LP, suggesting that T5 is using #rule as a statistical feature to make predictions.

C Algorithms: Rule-Priority & Label-Priority

a *Rule-Priority* (RP)

```
pred_num ~ U[5, 30]
preds ← Sample(vocab, pred_num)
fact_num ~ U[1, pred_num]
rule_num ~ U[0, 4 * pred_num]
rules ← empty array
while size of rules < rule_num do
    body_num ~ U[1, 3]
    body ← Sample(preds, body_num)
    head ← Sample(preds, 1)
    if tail ∉ body then
        add body → head to rules
    end if
end while
fact_num ~ U[0, pred_num]
facts ← Sample(preds, fact_num)
query ← Sample(preds, 1)
Compute label via forward-chaining.
return (facts, rules, query, label)
```

b *Label-Priority* (LP)

```
pred_num ~ U[5, 30]
preds ← Sample(vocab, pred_num)
rule_num ~ U[0, 4 * pred_num]
set l ~ U[1, pred_num/2] and group preds
into l layers
for predicate p in layer 1 ≤ i ≤ l do
    q ~ U[0, 1]
    assign label q to predicate p
    if i > 1 then
        k ~ U[1, 3]
        cand ← nodes in layer i − 1
                with label = q
        body ← Sample(cand, k)
        add body → p to rules
    end if
end for
while size of rules < rule_num do
    body_num ~ U[1, 3]
    body ← Sample(preds, body_num)
    head ← Sample(preds, 1)
    add body → tail to rules unless tail has label 0 and
    all predicates in body has label 1.
end while
facts ← predicates in layer 1 with label = 1
query ← Sample(preds, 1)
label ← pre-assigned label for query
return (facts, rules, query, label)
```

Figure 9: Two sampling algorithms Rule-Priority and Label-Priority. $Sample(X, k)$ returns a random subset from X of size k. $U[X, Y]$ denotes the uniform distribution over the integers between X and Y.

D Proof of Theorem 1

We prove theorem 1 by construction: in N-layer BERT model, we take the first layer as parsing layer, the last layer as output layer and the rest layers as forward chaining reasoning layer. Basically, in the parsing layer we preprocess the natural language input. In forward chaining reasoning layers, the model iteratively broadcast the RHSs to all LHSs, and check the left hand side (LHS) of each rule and update the status of the right hand side (RHS). Here we introduce the general idea of the construction, and we will release the source code for the detailed parameters assignments.

D.1 Pre-processing Parameters Construction

Predicate Signature For each predicate P, we generate its signature $Sign_P$, which is a 60-dimensional unit vector, satisfying that for two different predicates P_1, P_2, $Sign_{P_1} \cdot Sign_{P_2} < 0.5$. We can randomly generate those vectors and check until the constraints are satisfied. Empirically it takes no more than 200 trials.

Meaningful Vector In parsing layer, we process the natural language inputs as multiple "meaningful vectors". The meaningful vectors are stored in form of $L_A||L_B||L_C||R||0^{512}$, representing a rule $L_A \wedge L_B \wedge L_C \to R$. Each segment L_A, L_B, L_C, R has 64 dimensions, representing a predicate or a always True/False dummy predicate. For each predicate P, the first 63 dimensions, denoted as P^{sign}, form the signature of the predicate, and the last dimension is a Boolean variable, denoted as P^v. The following information is converted into meaningful vectors:

1. Rule $LHS \to RHS$: if the LHS has less than 3 predicates, we make it up by adding always True dummy predicate(s), and then encode it into meaningful vector, stored in the separating token

follows the rule. In addition, for each predicate P in LHS, we encode a dummy meaningful vector as $False \to P$ and store it in the encoding of P. This operation makes sure that every predicate in the input sentence occurs at least once in RHS among all meaningful vectors. We will see the purpose later.

2. Fact P: we represent it by a rule $True \to P$, and then encode it into meaningful vector and store it in the embedding of the separating token follows the fact.

3. Query Q: we represent it by a rule $Q \to Q$, encode and store it in the [CLS] token at beginning.

Hence, in the embedding, some positions are encoded by meaningful vectors. For the rest positions, we use zero vectors as their embeddings.

D.2 Forward Chaining Parameters Construction

Generally, to simulate the forward chaining algorithm, we use the attention process to globally broadcast the true value in RHSs to LHSs, and use the MLP layers to do local inference for each rule from the LHS to the RHS.

In attention process, for each meaningful vector, the predicates in LHS look to the RHS of others (including itself). If a RHS has the same signature as the current predicate, the boolean value of the RHS is added to the boolean value of the current predicate. Specifically, we construct three heads. We denote $Q_i^{(k)}$ to stand for the query vector of the i-th token of the k-th attention head. For a meaningful vector written as $L_A||L_B||L_C||R||0^{512}$,

$$
\begin{aligned}
&Q_i^{(1)} = L_A^{sign}||\frac{1}{4}, Q_i^{(2)} = L_B^{sign}||\frac{1}{4}, Q_i^{(1)} = L_C^{sign}||\frac{1}{4} \\
&K_i^{(1)} = \beta R, K_i^{(2)} = \beta R, K_i^{(3)} = \beta R \\
&V_i^{(1)} = 0^{63}||R^v, V_i^{(2)} = 0^{63}||R^v, V_i^{(3)} = 0^{63}||R^v.
\end{aligned}
$$

Here β is a pre-defined constant. The attention weight to a different predicate is at most $\frac{3\beta}{4}$, while the attention weight to the same predicate is at least β, and the predicate with positive boolean value has even larger ($\frac{5\beta}{4}$) attention weight. Thus, with a large enough constant β, we are able to make the attention distribution peaky. Theoretically, when $\beta > 300 \ln 10$, we can guarantee that the attention result

$$
Attention(Q, K, V) = softmax\left(\frac{QK^T}{\sqrt{d_k}}\right) V
$$

satisfies that the value is in the range of $[0.8, 1.0]$ if the predicate on LHS is boardcasted by some RHS with true value, otherwise it is in the range of $[0, 0.2]$.

This attention results are added to the original vectors by the skipped connection. After that, we use the two-layer MLP to do the local inference in each meaningful vector. Specifically, we set

$$
\begin{aligned}
10[&ReLU(L_A^v + L_B^v + L_C^v - 2.3) \\
&-ReLU(L_A^v + L_B^v + L_C^v - 2.4)]
\end{aligned}
$$

as the updated R^v. Thus, $R^v = 1$ if and only if all the boolean values in LHS are true, otherwise $R^v = 0$. We also set L_A^v, L_B^v, L_C^v as 0 for the next round of inference.

D.3 Output Layer Parameters Construction

In output layer, we take out the Boolean value of the RHS of the meaningful vector in [CLS] token.

E Training Details

When training BERT on RP or LP, we train for 20 epochs with a learning rate of 4×10^{-5}, a warm-up ratio of 0.05, and a batch size of 64. Training takes approximately 2 days on 4 NVIDIA 1080Ti / 2080Ti GPUs with 12Gb GPU memory. When training T5-base, we use the same learning rate and batch size and train for the same number of epochs.

F Examples from SimpleLogic

Rules: If messy and hypocritical and lonely, then shiny. If tame, then friendly. If plain and shiny and homely, then nervous. If tender, then hypocritical. If dull and impatient and plain, then tame. If spotless, then perfect. If elegant and tender, then homely. If lonely and inquisitive and plain, then homely. If proud, then quaint. If outrageous and homely and impatient, then messy. If quaint, then outrageous. If elegant and glamorous and ugly, then homely. If perfect and sincere and mean, then ambitious. If spotless and quaint and tame, then messy. If tame and sincere and homely, then elegant. If ambitious, then elegant. If shiny and proud, then combative. If quaint and elegant and nervous, then impatient. If glamorous, then outrageous. If proud, then friendly. If combative and nervous, then outrageous. If outrageous and quaint, then careless. If lonely and plain, then inquisitive. If lonely and ugly and combative, then tame. If friendly, then dull. If lonely, then tame. If tender and plain and lonely, then elegant. If glamorous, then hypocritical. If tame and helpless and impatient, then friendly. If careless and messy, then nervous. If combative and shiny, then inquisitive. If plain and outrageous and ugly, then glamorous. If careless and quaint and spotless, then combative. If homely, then helpless. If ambitious, then proud. If messy and ugly, then inquisitive. If perfect, then proud. If helpless and perfect, then elegant. If perfect, then lonely. If lonely and hypocritical, then perfect. If perfect, then friendly. If tender and messy and ambitious, then quaint. If proud, then mean. If outrageous, then perfect. If nervous, then inquisitive. If hypocritical and homely and nervous, then tender. If friendly and dull and outrageous, then ambitious. If glamorous, then proud. If impatient and nervous, then spotless. If mean and quaint and lonely, then spotless. If glamorous, then careless. If dull and mean, then elegant. If homely, then proud. If inquisitive and plain, then ugly. If tender, then homely. If proud and quaint and lonely, then outrageous. If glamorous and perfect and dull, then messy. If helpless and tame and tender, then proud. If friendly and mean, then helpless. If inquisitive, then spotless. If shiny, then tame. If perfect and quaint, then careless. If careless and nervous and combative, then homely. If outrageous and inquisitive and elegant, then hypocritical. If tender and quaint and perfect, then careless. If mean and friendly and ambitious, then combative.
Facts: Alice shiny. Alice tender. Alice lonely.
Query: Alice is dull ?
Label: True
Proof Depth: 3
From: RP

Rules: If witty, then diplomatic. If careless and condemned and attractive, then blushing. If dishonest and inquisitive and average, then shy. If average, then stormy. If popular, then blushing. If talented, then hurt. If popular and attractive, then thoughtless. If blushing and shy and stormy, then inquisitive. If adorable, then popular. If cooperative and wrong and stormy, then thoughtless. If popular, then sensible. If cooperative, then wrong. If shy and cooperative, then witty. If polite and shy and thoughtless, then talented. If polite, then condemned. If polite and wrong, then inquisitive. If dishonest and inquisitive, then talented. If blushing and dishonest, then careless. If inquisitive and dishonest, then troubled. If blushing and stormy, then shy. If diplomatic and talented, then careless. If wrong and beautiful, then popular. If ugly and shy and beautiful, then stormy. If shy and inquisitive and attractive, then diplomatic. If witty and beautiful and frightened, then adorable. If diplomatic and cooperative, then sensible. If thoughtless and inquisitive, then diplomatic. If careless and dishonest and troubled, then cooperative. If hurt and witty and troubled, then dishonest. If scared and diplomatic and troubled, then average. If ugly and wrong and careless, then average. If dishonest and scared, then polite. If talented, then dishonest. If condemned, then wrong. If wrong and troubled and blushing, then scared. If attractive and condemned, then frightened. If hurt and condemned and shy, then witty. If cooperative, then attractive. If careless, then polite. If adorable and wrong and careless, then diplomatic. Facts: Alice sensible Alice condemned Alice thoughtless Alice polite Alice scared Alice average
Query: Alice is shy ?
Label: False
Proof Depth: 3
From: RP

Rules: If comfortable, then tense. If nervous, then blushing. If nervous and difficult, then beautiful. If disgusted, then clean. If talkative and aggressive, then light. If versatile and supportive, then beautiful. If aggressive, then different. If glamorous and supportive and pleasant, then inexpensive. If light and outrageous and modern, then pleasant. If blushing, then tense. If beautiful, then clean. If perfect and inexpensive, then comfortable. If modern and different, then supportive. If tense, then glamorous. If talkative and aggressive and perfect, then blushing. If versatile, then outrageous. If tense, then perfect. If modern and perfect and inexpensive, then difficult. If versatile and aggressive, then reserved. If comfortable and versatile, then modern. If pleasant and versatile, then reserved. If clean and tense and difficult, then outrageous. If glamorous and modern, then courageous. If elegant and clean, then perfect. If pleasant, then tense. If versatile and blushing and elegant, then light. If reserved, then clean. If clean and talkative and difficult, then reserved. If light, then courageous. If blushing, then light. If different and beautiful, then modern. If disgusted and talkative, then perfect. If elegant and reserved and talkative, then aggressive. If elegant and courageous, then outrageous. If modern and difficult, then disgusted. If supportive and beautiful, then light. If blushing, then glamorous. If comfortable and modern and glamorous, then blushing. If disgusted and inexpensive and talkative, then difficult. If different and clean and disgusted, then modern. If clean and talkative and light, then supportive. If modern and nervous, then difficult. If talkative and aggressive, then modern. If tense and beautiful, then supportive. If modern and inexpensive and glamorous, then comfortable. If difficult and beautiful and modern, then supportive. If nervous and elegant and aggressive, then modern. If tense, then light. If comfortable and inexpensive and disgusted, then tense. If inexpensive and elegant, then nervous. If nervous, then elegant. If glamorous and pleasant, then elegant. If elegant and outrageous, then pleasant. If aggressive and disgusted and comfortable, then light. If talkative and reserved, then clean. If aggressive and modern and inexpensive, then supportive. If reserved and versatile and glamorous, then modern. If comfortable and pleasant and beautiful, then outrageous. If nervous and different and elegant, then modern. If difficult and perfect and outrageous, then tense. If comfortable and blushing and glamorous, then clean. If disgusted, then inexpensive. If inexpensive and tense, then blushing. If elegant, then aggressive. If inexpensive and versatile, then pleasant. If supportive and tense and beautiful, then disgusted. If glamorous and beautiful, then talkative. If tense and reserved, then beautiful. If different, then pleasant. If glamorous and supportive, then clean.
Facts: Alice versatile. Alice beautiful. Alice light. Alice glamorous. Alice outrageous. Alice difficult.
Query: Alice is comfortable ?
Label: True
Proof Depth: 6
From: RP

Rules: If attentive and loving and beautiful, then helpful. If bad-tempered and nervous, then dull. If unpleasant and elated and proud, then gifted. If easy and ugly and unpleasant, then frantic. If courageous and dull and nervous, then loving. If gifted, then nervous. If unpleasant, then bad-tempered. If easy and excited, then unpleasant. If impartial and gifted, then helpful. If shy and elated and courageous, then excited. If stubborn, then straightforward. If thoughtless, then excited. If beautiful and stubborn and straightforward, then bossy. If anxious and ugly and courageous, then elated. If thoughtless and loving and impartial, then courageous. If beautiful and stubborn and loving, then dull. If impartial and shy, then frantic. If thoughtless and excited, then condemned. If helpful and beautiful and shy, then bossy. If ambitious and frantic, then straightforward. If condemned, then easy. If nervous, then loving. If attentive and helpful and beautiful, then condemned. If easy, then nervous. If impartial and frantic and bad-tempered, then attentive. If condemned and stubborn, then elated. If anxious and ugly, then excited. If stupid, then nervous. If thoughtless and stupid and courageous, then condemned. If straightforward and shy and loving, then stupid. If courageous and anxious and gifted, then elated. If unpleasant and beautiful and condemned, then stubborn. If frantic, then straightforward. If attentive, then bad-tempered. If unpleasant, then bossy. If bossy and courageous, then straightforward. If nervous and condemned, then courageous. If ambitious and elated and bad-tempered, then ugly. If beautiful, then loving. If ambitious and frantic, then easy. If helpful and unpleasant and excited, then beautiful. If courageous, then shy. If loving and bad-tempered, then proud. If anxious, then bad-tempered. If elated and anxious and bad-tempered, then courageous. If ambitious, then bossy. If ambitious and helpful, then excited. If shy and easy and stupid, then helpful. If helpful and unpleasant and thoughtless, then shy. If elated and gifted and easy, then anxious. If helpful and ambitious and condemned, then easy. If stubborn and proud, then bad-tempered. If stubborn and thoughtless and attentive, then unpleasant. If stupid and elated, then bossy. If stubborn and attentive and impartial, then straightforward. If attentive, then thoughtless. If loving and ambitious, then dull. If unpleasant and thoughtless and courageous, then straightforward. If bad-tempered and stubborn, then easy. If bad-tempered, then excited. If impartial, then proud. If impartial, then unpleasant. If bossy and proud and attentive, then condemned. If helpful and nervous and bad-tempered, then easy. If beautiful, then excited. If attentive and straightforward, then proud. If shy and impartial, then unpleasant. If thoughtless, then easy. If easy and beautiful and proud, then bossy. If bossy and condemned and proud, then dull. If thoughtless and attentive and anxious, then helpful. If dull, then proud. If ugly and gifted and ambitious, then beautiful. If proud, then frantic. If thoughtless and stupid and shy, then impartial. If condemned and excited, then stubborn. If straightforward and impartial, then frantic.
Facts: Alice gifted. Alice ambitious. Alice stupid.
Query: Alice is anxious ?
Label: False
Proof Depth: 6
From: RP

Rules: If blushing and disgusted, then fancy. If impatient, then long. If frantic, then long. If blushing and frail, then gifted. If frail and long and fancy, then disgusted. If frantic and helpless, then gifted. If broad-minded and frantic, then blushing. If helpless, then broad-minded. If frantic and disgusted and frail, then blushing. If helpless, then impatient. If blushing, then disgusted. If long and gifted and blushing, then frantic. If frantic, then blushing. If fancy, then impatient. If gifted, then fancy. If frail, then helpless. If blushing and frail, then helpless. If blushing, then gifted. If broad-minded and impatient, then long. If broad-minded and disgusted, then fancy. If impatient and disgusted and long, then broad-minded. If broad-minded, then helpless. If disgusted and gifted, then blushing. If gifted and frantic, then fancy. If frail, then broad-minded. If fancy, then broad-minded. If broad-minded, then helpless. If blushing and disgusted, then fancy. If frantic and blushing and gifted, then frail. If frantic, then disgusted. If disgusted, then fancy. If fancy and helpless, then frantic. If frail and disgusted and helpless, then broad-minded. If frantic, then gifted. If long and fancy, then frantic. If blushing, then gifted. If impatient and helpless and gifted, then frantic. If frail and gifted and impatient, then broad-minded. If helpless, then broad-minded.
Facts: Alice frail.
Query: Alice is disgusted ?
Label: False
Proof Depth: 3
From: LP

Rules: If different and disobedient, then witty. If agreeable, then weary. If aggressive, then elated. If ugly, then serious. If aggressive and enchanting and frail, then rational. If rude and serious and pessimistic, then ugly. If talented and aggressive and busy, then disobedient. If aggressive and weary and victorious, then serious. If weary and witty and talented, then different. If straightforward, then victorious. If rational and aggressive and disobedient, then tidy. If wrong and serious, then agreeable. If rude, then talented. If rational and tense and rude, then aggressive. If wrong, then stormy. If tense, then wrong. If elated and talented, then enchanting. If rude and weary, then ugly. If tidy, then elated. If tidy and talented, then calm. If long and weary, then wrong. If serious, then weary. If tense and rational and agreeable, then victorious. If agreeable and different, then enchanting. If weary and straightforward, then agreeable. If wandering, then stormy. If rude, then stormy. If shiny, then rational. If rational and serious, then straightforward. If wrong, then wandering. If agreeable and aggressive and rude, then shiny. If victorious, then serious. If ugly and rude, then tidy. If different, then wandering. If agreeable and weary and long, then wandering. If witty and frail and aggressive, then different. If enchanting, then exuberant. If busy and aggressive, then pessimistic. If talented and ugly, then exuberant. If rude, then victorious. If elated and calm, then shiny. If frail, then long. If straightforward and ugly and victorious, then calm. If exuberant and tidy, then victorious. If wrong and talented, then aggressive. If shiny and pessimistic and busy, then tense. If agreeable and tidy, then rude. If witty and wandering, then busy. If exuberant and ugly and frail, then pessimistic. If busy and long and calm, then frail. If stormy and calm, then straightforward. If shiny and wrong and frail, then wandering. If long and agreeable, then stormy. If long and tidy, then talented. If exuberant, then different. If rude, then disobedient. If tense and long, then witty. If witty, then pessimistic. If agreeable and tidy and weary, then wrong. If talented and busy, then straightforward. If long and aggressive, then exuberant. If shiny and tidy and witty, then rude. If exuberant, then disobedient. If straightforward and weary, then aggressive. If aggressive, then different. If frail, then tense. If calm and elated, then victorious. If long and tense, then enchanting. If calm and ugly, then aggressive.
Facts: Alice frail.
Query: Alice is stormy ?
Label: True
Proof Depth: 3
From: LP

Rules: If frantic and helpful, then victorious. If inquisitive and zealous, then bad-tempered. If busy and vivacious, then condemned. If embarrassed, then rude. If thoughtful and rude and helpful, then zealous. If agreeable, then curious. If witty and perfect and thoughtful, then shiny. If impartial and tense, then fine. If frantic and thoughtful and busy, then embarrassed. If agreeable, then pessimistic. If busy and long and embarrassed, then thoughtful. If long and intellectual and fancy, then enchanting. If perfect and victorious and hurt, then zealous. If inquisitive and hurt, then vivacious. If disgusted and tense, then intellectual. If fine, then busy. If fancy and bad-tempered, then fine. If thoughtful, then long. If victorious and condemned, then hurt. If tense, then fine. If frantic, then enchanting. If victorious, then impartial. If agreeable, then enchanting. If hurt and zealous and inquisitive, then fancy. If curious, then frantic. If helpful and zealous, then intellectual. If busy and curious, then agreeable. If curious, then helpful. If curious and victorious, then pessimistic. If witty and shiny and busy, then perfect. If rude and condemned and victorious, then zealous. If witty and embarrassed, then frantic. If perfect and victorious and enchanting, then fancy. If zealous and witty, then rude. If hurt and curious and condemned, then embarrassed. If victorious and busy and disgusted, then intellectual. If fancy and shiny, then enchanting. If hurt and victorious and agreeable, then curious. If thoughtful and helpful, then disgusted. If fancy and intellectual, then shiny. If frantic and impartial, then embarrassed. If impartial, then thoughtful. If pessimistic, then curious. If condemned, then thoughtful. If enchanting, then witty. If zealous and inquisitive and agreeable, then condemned. If fancy and inquisitive, then bad-tempered. If enchanting and fancy and rude, then curious. If vivacious and condemned, then zealous. If perfect, then impartial. If helpful and embarrassed and frantic, then condemned. If helpful, then perfect. If curious, then embarrassed. If condemned, then enchanting. If fine and intellectual, then shiny. If hurt and agreeable, then victorious. If victorious and condemned and rude, then inquisitive. If fancy, then victorious. If impartial and frantic and curious, then hurt. If fancy and long, then vivacious. If hurt and vivacious, then tense. If witty and vivacious and helpful, then embarrassed. If curious, then hurt. If fancy and rude, then zealous. If impartial and shiny and rude, then tense. If pessimistic, then embarrassed. If disgusted and busy and rude, then long. If witty and embarrassed and victorious, then pessimistic. If curious and agreeable, then vivacious. If embarrassed and hurt, then victorious. If intellectual, then witty.
Facts: Alice tense. Alice disgusted.
Query: Alice is hurt ?
Label: False
Proof Depth: 6
From: LP

Rules: If tense and tame, then rude. If disgusted, then stormy. If modern and dishonest, then tense. If light, then inquisitive. If disgusted, then light. If elegant, then average. If bright, then ugliest. If versatile and average and tense, then stormy. If tense and disobedient, then powerful. If ugliest, then careless. If nervous and ugliest, then outstanding. If versatile, then tense. If lonely and helpless, then modern. If popular, then powerful. If worrisome and ugly and ugliest, then elegant. If worrisome and helpless and popular, then alert. If attractive, then disgusted. If modern and dishonest, then disobedient. If careless and bright, then elegant. If disgusted, then helpless. If attractive and ugliest, then stormy. If careless and alert, then stormy. If careless, then attractive. If cruel and versatile and ugly, then dishonest. If disobedient and elegant and ugly, then nervous. If popular, then ugly. If bright and lonely, then elegant. If alert, then rude. If versatile, then alert. If versatile and helpless, then stormy. If popular and lonely, then rude. If ugly, then stormy. If alert and bright, then worrisome. If ugly, then bright. If ugly and lonely, then helpless. If tense and disgusted, then alert. If outstanding, then inquisitive. If disobedient, then elegant. If careless and alert and ugly, then dishonest. If rude and lonely and powerful, then average. If ugly and versatile and helpless, then worrisome. If worrisome, then popular. If powerful and dishonest and ugly, then versatile. If disgusted, then average. If cruel, then light. If outstanding and bright and cruel, then stormy. If powerful, then tense. If disobedient and tense, then lonely. If tame, then cruel. If inquisitive and lonely, then outstanding. If lonely, then outstanding. If ugly, then alert. If nervous and careless, then worrisome. If disobedient, then powerful. If helpless, then careless. If popular and average, then versatile. If helpless, then ugliest. If light, then rude. If ugly and stormy and disgusted, then nervous. If average and ugly and attractive, then nervous. If worrisome and stormy and careless, then bright. If ugly and popular and attractive, then helpless. If dishonest and rude and helpless, then popular. If tense and bright and disgusted, then ugliest. If careless and ugliest, then stormy. If nervous and tense and quaint, then cruel. If versatile, then helpless. If alert, then attractive. If ugliest, then nervous. If popular, then inquisitive. If helpless, then disgusted. If tame, then quaint. If inquisitive, then dishonest. If careless and nervous, then versatile. If alert, then attractive. If lonely and helpless, then elegant. If quaint, then outstanding. If modern and versatile, then stormy. If quaint and ugliest and popular, then dishonest. If bright and stormy, then attractive. If inquisitive and rude, then modern. If popular, then powerful. If elegant, then average. If helpless and average and lonely, then cruel. If bright, then ugliest. If average and helpless, then nervous. If tame and popular, then powerful. If rude and disobedient, then elegant. If ugly and disgusted, then nervous. If worrisome, then rude. If ugliest, then cruel. If versatile and ugly and careless, then cruel. If outstanding, then elegant. If quaint and attractive, then careless. If nervous, then powerful. If ugliest, then rude. If elegant and outstanding, then rude. If disobedient and dishonest, then modern. If worrisome and ugliest, then versatile. If alert, then helpless. If modern and tame and outstanding, then disobedient. If modern and popular, then disobedient. If light, then rude.
Facts: Alice tame.
Query: Alice is powerful ?
Label: True
Proof Depth: 6
From: LP